The Arithmetic Teacher

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A Comparison of Achievement in Arithmetic in England and Central California

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IN MAY 1955, an arithmetic test containing 100 items was administered to a cross-section sample of schools in England under the direction of the British National Foundation for Educational Research. Through arrangements with the Foundation and the University of California (Berkeley) an adapted form of the same test* was given to a similarly selected sample of schools in Central California in January, 1957. The study was financed by the research fund of the Department of Education of the University of California (Berkeley). This article reports a comparison of the results.

The Test

The test used was printed in two sections. Section I contained 40 non-verbal computations and Section II consisted of 60 verbal exercises and problems. The time limits for the two sections were respectively 20 minutes and 30 minutes, with a short interval between. In adapting the test for American usage certain changes were necessary. (a) Seventy of the 100 items were appropriate for use here with only minor verbal changes. (b) Twenty-six items involved money terms. These were changed to dollars and cents, with due regard to size of numbers and the

arithmetical processes involved. These items were scored separately and will be reported at a later time. (c) The remaining four items required conversion from weight in "stones" to pounds, and results from these four were not tabulated. The data for the present report are derived from the 70 comparable items, of which 31 were computational exercises in Section I and 39 were verbal exercises from Section II. The reliability of the test was .89 for the total of 70 items, and was .68 for Section I and .87 for Section II separately, using Kuder-Richardson formula 20 with the 544 papers that were item analyzed.

The Sample Tested

Because of differences in school organization in England and the United States, a more valid comparison can be made by keeping the age-range constant than by trying to compare corresponding school grades. Accordingly, the test in both countries was limited to the age-range of 10 yr. 8 mo. to 11 yr. 7 mo. as of the month in which the test was given. The directions to the California schools were to include "all pupils enrolled in your schools whose age is from 10 yr. 8 mo. to 11 yr. 7 mo. as of January 1957. This will include all pupils who were born between May 1, 1945 and April 30, 1946 inclusive." The test was given to all such pupils in attendance on the day the test was administered.

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* "Arithmetic Test 5" (Test Publication No. 45) of the National Foundation for Research in Education in England and Wales, 79 Wimpole St., London W.I.

In England the test was given to schools selected by the technique of "random numbers sampling" and the same method was followed here. The 773 urban and 661 rural schools in 21 counties in Central California were numbered serially. Then, using Tippit's *Tables of Random Numbers*, schools were selected "randomly" to provide a total sample of approximately 3000 cases, the size of the sample in England. The sample was stratified according to urban and rural schools keeping the proportion of cases approximately the same as in the total population, as was done in England. Invitations were issued to this randomly selected group of schools to participate in the experiment. Only three of the schools invited refused— two small cities, because they were already involved in an arithmetic survey, and San Francisco, because due to "requests from five colleges and universities having graduate schools, it is virtually impossible to authorize so many interruptions of the instructional program." The final sample was composed of 70 schools, of which 39 were urban and 31 rural. Minor exceptions to the sampling procedure were as follows: Three urban schools included only grades 1 to 5 and consequently did not cover the 10– 11 year age range, so schools which had six grades from the same cities were substituted; two schools selected had been closed and two others in the same system were selected in their places; in one city, through misunderstanding, both of its two elementary schools were included instead of only the one selected. Three exceptions occurred in the rural group: One school had no pupils in the age range; in one school the teacher had suffered a severe accident and a substitute school (7 pupils) was included; one small school of less than 5 pupils in the age range was dropped because of distance to be travelled (400 miles). The final sample for the study included 3179 pupils, of which 2483 were urban and 696 rural. Since enrollments in English schools are generally smaller than here, their sample was drawn from 46 urban and 45 rural schools, a total

of 91 as compared with 70 here. The data on the sample are summarized in Table 1.

Administration of the Tests

All of the tests were administered by the writer (33 schools) or by his research assistant, Mr. George Yonge (37 schools). Generally one or more teachers or supervisors were also in the room during the testing. The directions, printed on the first page of the test, were given orally and were uniform with those used in England. In some of the schools the test was given in the regular classrooms, pupils outside the age-range being sent to other parts of the building, while in the larger schools the pupils to be tested were assembled in a multi-purpose room using cafeteria or library tables. The physical conditions for testing were good. In the interval between Sections I and II of the test, the pupils stood, stretched and talked informally. The tests were timed with stop-watches.

TABLE 1

NUMBER OF SCHOOLS AND PUPILS IN THE SAMPLES
FROM ENGLAND AND CENTRAL CALIFORNIA

Sample tested	England	California
Number of schools:		
Urban	46	39
Rural	45	31
Total	91	70
Number of pupils:		
Urban	2703	2483
Rural	488	696
Total	3191	3179
Per cent urban pupils in sample tested*	85%	78%
Per cent urban population in area sampled	82%	73%

* The definition and classification of urban and rural in England is not precisely the same as in California; hence, differences should not be interpreted rigidly.

Prior to the time of testing, lists were received from each school giving the names and birth dates for all pupils in the age range. At the time of testing each pupil also wrote his birth date opposite his name on the test blank, thus giving a double check on

age. All tests were scored by clerical assistants in the research office at the University. Their work was checked systematically to assure correct scoring. The tests were given during the period from January 8 to February 8, at the end of the semester, with the exception of three small rural schools which were tested a week later.

The reader should keep in mind that the two groups in England and California were uniform in chronological age rather than in school grade. Difference in promotion rates, school organization, placement of curriculum materials, and such factors naturally vary from country to country. The purpose of this investigation is to compare the achievement in arithmetic in England and California of sizable samples of pupils of the same chronological age. The outcome will be simply a body of facts. These facts must then be evaluated in terms of the total philosophy and objectives of education in the two areas that were studied. Final answers are, of course, matters for public decision, but such decisions can be intelligent only if they take account of factual evidence.

Comparison of Results

The results of the tests are shown in Table 2, which gives the means and standard deviations for the scores from England and California, and in Table 3, which shows the distribution of scores. As indicated in Table 2, the mean scores on the total test (70 comparable items) were 29.1 for the English schools and 12.1* for those in California. In the computation part of test (Section 1) the mean scores were 14.1 for England and 4.2 for California, while in the verbal problems part (Section 2) the corresponding means were 15.0 and 7.8. As would be expected, the urban schools scored higher than the rural schools in both countries. The differences between the scores are all statistically significant at well above the 1 per cent level.

* There is a 99% probability that the mean for the total school population from which the sample was drawn will lie within the interval 10.0 to 14.2.

TABLE 2

RAW SCORES ON ARITHMETIC TEST FOR 70 COMPARABLE ITEMS IN RANDOM SAMPLE OF 91 SCHOOLS IN ENGLAND AND 70 SCHOOLS IN CENTRAL CALIFORNIA. (NUMBER OF PUPILS: ENGLAND 3191; CALIFORNIA 3179.)

| | England | | California | | Difference* |
	Mean	S.D.	Mean	S.D.	
Sec. 1, Computation					
Urban	14.3	8.8	4.3	2.1	10.0
Rural	12.8	8.4	4.0	2.2	8.8
Total	14.1	8.7	4.2	2.1	9.9
Sec. 2, Problems					
Urban	15.3	10.6	8.2	5.2	7.1
Rural	13.2	10.0	6.7	4.7	6.5
Total	15.0	10.5	7.8	5.1	7.2
Total test					
Urban	29.6	18.8	12.5	6.8	17.1
Rural	26.0	17.6	10.7	6.5	15.3
Total	29.1	18.7	12.1	6.8	17.0

* All differences are significant at 1 per cent level.

TABLE 3

DISTRIBUTION OF TOTAL SCORES ON 70 ITEMS ARITHMETIC TEST (BOTH SECTIONS) IN ENGLAND AND CALIFORNIA

| Raw Score | Urban Schools | | Rural Schools | | All Schools | |
	England	Calif.	England	Calif.	England	Calif.
69–70	7	0	0	0	7	0
66–68	40	0	3	0	43	0
63–65	57	0	5	0	62	0
60–62	73	0	1	0	74	0
57–59	95	0	16	0	111	0
54–56	117	0	14	0	131	0
51–53	99	1	12	0	111	1
48–50	114	2	14	0	128	2
45–47	126	2	22	2	148	4
42–44	110	0	28	1	138	1
39–41	106	5	18	0	124	5
36–38	97	6	25	1	122	7
33–35	116	13	33	3	149	16
30–32	115	27	16	1	131	28
27–29	106	34	22	11	128	45
24–26	142	56	23	10	165	66
21–23	127	111	20	15	147	126
18–20	135	205	23	44	158	249
15–17	152	328	23	61	175	389
12–14	161	443	32	112	193	555
9–11	170	559	32	160	202	719
6– 7	197	384	33	139	230	523
3– 5	171	218	45	99	216	317
0– 2	70	89	28	37	98	126
n	2703	2483	488	696	3191	3179

A more detailed picture of the differences between the two groups of pupils is given in Table 3. The two columns at the right of the table give the data for urban and rural schools combined. As indicated, the scores for the English pupils spread over the entire range of the test from zero to a perfect score of 70, whereas there were no scores above the interval 51–53 for the pupils from California. Thirteen pupils in California made scores above 38 as compared with 1077 pupils in England who scored above this point. There were some zero scores in both groups, 23 in California, and 13 in England. It should be recalled that the tests were given to all pupils in the schools who fell in the age-range covered, and therefore included a few who were definitely at a subnormal level.

The test used in this study was constructed in England and was used because scores from an excellent nationwide sample of their schools were already available. The test gave more emphasis in Section 1 to denominate numbers than is usual in American tests. Therefore, an item analysis of the results was made from which one can see the differences in scores for each of the 70 items. This item analysis is based on a random selection of 456 test papers in England and 544 in California. The results are shown in Tables 4 and 5.

Table 4 should be read as follows: on the first item in Section 1, the correct answer was obtained by 93% of the sample from England and 85% of the sample from California. It will be noted that the English averages were above those in California in all of the 31 items in Section 1 and in all except 2 items (No. 2 and No. 5) in Section 2. There were 5 items in the test which involved hundredweight (cwt.). Although this term is included in the tables of the California state textbook there was no indication that it had been taught and, furthermore, it is seldom used in America whereas it is a commonly used measure in England. The average per cent correct on these 5 items (nos. 22, 27, 35, and 36 in Sec. 1 and No. 21

TABLE 4
ITEM ANALYSIS OF 70 COMPARABLE EXERCISES SHOWING PER CENT OF TOTAL NUMBER OF PUPILS WHO DID EACH EXERCISE CORRECTLY. (SAMPLE OF 456 PUPILS IN ENGLAND AND 544 PUPILS IN CALIFORNIA)

Item Number*	Per Cent Correct		Item Number	Per Cent Correct	
	Eng-land	Cali-fornia		Eng-land	Cali-fornia
(Sec. 1)			(Sec. 2)		
1	93%	85%	1	91%	89%
2	84	80	2	84	85
3	76	67	3	73	65
4	78	59	5	59	66
5	61	7	6	69	55
6	70	66	9	54	23
7	59	4	12	46	42
8	58	0	13	61	15
9	64	2	15	37	13
10	49	3	16	48	24
11	48	2	17	51	19
12	57	1	18	34	4
14	62	4	20	43	21
16	49	0	21	40	7
17	57	2	22	47	33
18	47	1	23	52	39
19	43	22	24	44	28
22	45	0	25	45	7
23	44	1	29	47	7
25	42	3	30	34	6
27	37	0	32	47	7
28	31	0	33	45	9
30	24	0	34	34	7
31	26	0	36	33	19
32	22	0	37	31	17
33	22	0	41	29	4
34	17	1	42	30	7
35	18	0	44	21	7
36	15	0	45	24	3
37	14	0	46	23	5
39	9	4	47	22	3
			48	25	7
			49	12	1
			52	20	5
			55	11	1
			56	21	1
			57	13	1
			59	9	1
			60	3	1

* Item numbers correspond to the numbers on the printed test form.

in Sec. 2) is 31% in England and 1.4% in California. If these 5 items were omitted from the test it would, however, reduce the English mean for the total test in Table 2 by approximately 1.6 points but would make no significant difference in the comparison.

Table 5 gives some samples from the item analysis of the 70 comparable items in the test. The item numbers in both Tables 4 and 5 refer to the serial numbers on the printed test form. Table 5 should be read as follows: item No. 1, in Section 1, involved adding 4 two-place numbers in a vertical column. Ninety-three per cent of the English (E) pupils added correctly as compared to 85% of the California (C) pupils. In item 4, which called for dividing 987 by 7, 78% of the English pupils got the correct answer as compared to 59% for the California pupils. In exercises that involved denominate numbers the differences between the two groups was much more striking. In adding linear measures, as in Section 1 Item No. 5, the per cents correct in England and California were 61% and 7% respectively.

To summarize the factual results of the study, it is clear that pupils at age eleven in English schools are markedly superior to pupils of the same age in California in arithmetical achievement as measured by the 70 item test. In fact their mean score on the test was more than double the California mean, 29.1 as compared with 12.1.

TABLE 5

SAMPLE ITEMS AND PER CENTS OF ENGLISH AND CALIFORNIA SAMPLE DOING EACH CORRECTLY

Section One

1. Add E*—93%; C*—85%

    35
    47
    64
    23
    —

2. Subtract E—84%; C—80%

    537
    469
    —

3. Multiply E—76%; C—67%

    875
      6
    —

4. Divide E—78%; C—59%

    7)987

* The E and C prefixes before the per cents indicate England and California respectively.

5. Add E—61%; C—7%

ft.	in.
37	9
47	7
38	11

6. Subtract E—70%; C—66%

    8403
    7596
    —

7. Multiply E—59%; C—4%

lb.	oz.
7	4
	9

8. Divide E—58%; C—0%

tons	lb.
8)49	1200

10. Subtract E—49%; C—3%

gal.	pt.
86	4
47	5

11. Multiply E—48%; C—2%

yd.	ft.
15	2
	7

Section Two

1. E—91%; C—89%
   Add together 14, 5 and 9.
2. E—84%; C—85%
   John has 8 books, and I have four times as many. How many have I?
3. E—73%; C—65%
   Add 4 to 45 and divide the answer by 7.
5. E—59%; C—66%
   Underline the *smallest* fraction here and also write your answer on the line to the right.
   $\frac{1}{3}$ $\frac{1}{10}$ $\frac{1}{8}$ $\frac{1}{6}$ $\frac{1}{4}$ _____
6. E—69%; C—55%
   Write in figures the number ten thousand, one hundred and nine.
9. E—54%; C—23%
   A train leaves New York at 10 in the morning and arrives at 2 that afternoon. If it travels at 30 miles an hour, what is the length of the journey?
12. E—46%; C—42%
   Father was 42 years old in 1951. When was he born?
13. E—61%; C—15%
   What is half of $7\frac{1}{2}$?
15. E—37%; C—13%
   Write down the number with thirteen hundreds and seven units.
16. E—48%; C—24%
   A man rides a bicycle at 15 miles an hour. How long will he take to travel 90 miles at this rate?
17. E—51%; C—19%
   How many *minutes* are there between 9:30 A.M. and 10:45 A.M. the same morning?
18. E—34%; C—4%
   What is the cost of 1500 pounds of rock at $4 a ton?

One of the major factors that affects achievement in arithmetic is the content of the curriculum and the age or school grade at which the various topics are introduced. Here there are some sharp contrasts between practices in England and California. Although there is a considerable degree of freedom in individual schools as to the details of the curriculum, there is necessarily a certain amount of uniformity in sequence due to the nature of the number processes. For example, subtraction and multiplication must precede division because both are used in the division process. But there can be marked variations in the age or grade at which different processes, topics, and operations are introduced. California prints its own textbooks and hence an examination of the state adopted textbook in arithmetic gives a rather accurate picture of what is taught grade by grade. In England, individual schools select from a number of available series of textbooks. In order to make a factual comparison of school practices, the California state textbook and one of the textbooks used in England (Beacon Arithmetic by C. M. Fleming) were analyzed to find at which grade level each of the 70 items in the test used in this study was first introduced. For example, item No. 1 in the test was the addition of 4 two-digit numbers in column arrangement. This type of addition appeared first in the English book for their fourth school year and in the California book for Grade IV. However, it must be noted that in England children start at age 5 and begin the fourth school year at age 8, whereas in California children enter Kindergarten at age 5, grade one at age 6, and enter the fourth grade at age 9. Consequently, arithmetic, such as that in item No. 1 of the test, appears in the textbook for pupils of age 8 in England and of age 9 in California.

A comparison of the placement of the 70 items in the test in the two series of textbooks reveals the following facts. Eleven of the items in the test appeared in the English textbook for their fourth school year; 1 of these appeared in the California textbook for Grade III, 9 in the book for Grade IV, and 1 in the book for Grade V. The remaining 59 items in the test all appeared in the English textbook for their fifth school year; of these 29 appeared in the California fifth grade book and 30 not until the sixth grade book. Keeping in mind that pupils in the fourth year in English schools are one year younger than pupils in Grade IV here, a summary of the facts shows that one item in the test appeared in the textbook for pupils of the same age in the two countries, 37 items appeared in English textbooks for pupils one year younger than here, and 32 items in the books for pupils two years younger. These are facts of major importance in interpreting the results of the tests.

All of the 70 items in the test had been introduced in the English textbooks by the end of their fifth school year. Thirty of the items did not appear in the California books until Grade VI. Some of the pupils in the age range in California were still in Grade V and had not yet studied some of the items. The results of the tests do not indicate a lack of ability to learn arithmetic on the part of the children of California, but rather that the school program in arithmetic here is different from that in England.

Issues to be Studied

Although the primary purpose of this study was to secure some factual evidence regarding the relative achievement in arithmetic at age eleven in England and California, a serious attempt must also be made to think through the issues that are raised by the evidence. These matters are of concern to both the general public and to educators. Six issues related to the study are proposed for consideration.

1. There is no reason, *per se*, why the schools of California should be like the schools of England. Education is a large enterprise and arithmetic is only one of its parts. The desired outcomes in arithmetic must be decided in relation to other outcomes which are also desired. From the evi-

dence of this study, it is apparent that the communities of the schools sampled in California are accepting somewhat less than half of the achievement in arithmetic that is obtained in the English sample. It may well be that English schools are putting undue pressure on their pupils for achievement in school subjects, especially in preparation for their "11 year plus examination" which determines, for the most part, the type of secondary school the child will attend. It is a question of relative values. However, there is, as far as I can learn, no more evidence that emphasis on arithmetic is detrimental to personal-social development than there is that such emphasis is of positive value in giving substantial support to feelings of self-confidence and security. One may examine the per cents of pupils who secure correct answers to the 70 items of the test and then decide whether he is satisfied with the level of achievement that is indicated.

2. In attempting to interpret the evidence one might ask whether there are conditions peculiar to California which might have an adverse effect on achievement in arithmetic. Some such factors seem to be present. One of these is the very high degree of mobility of the population in California compared to the more stable population in England. For example, in two large schools included in the sample the change in school enrollment during the year amounted to more than 100%. This means that more than twice as many pupils were enrolled for varying lengths of time during the year than were enrolled at any one time. Part of this mobility is due to migratory agricultural workers, part to the prevailing tendency to move frequently from one location to another, and part to the continuous and large influx of population from other parts of the country. Persons unfamiliar with this degree of mobility fail to understand the problem of instruction which it places on the school. A second adverse factor is that of race and nationality. In two of the larger schools included, more than 98% of the pupils were negroes. The mean scores on the

seventy items for these schools were 6.2 and 6.6 compared to the mean of 12.1 for the total group of schools in the California sample. Judging as nearly as possible from names, there were 211 Mexicans in the sample who made a mean score of 9.3.

3. Age of beginning school is a third factor which might be examined. Since the English primary schools admit pupils at age five whereas the customary age for the first school grade here is six, it appears at first that English pupils have an advantage of an additional year of schooling by age eleven. However, more than 97% of the 3179 pupils in the California sample were from schools having a kindergarten which admits five year olds. The difference between the two groups, therefore, is not in the total number of years of school attendance but in the way the beginning years of school are used. Although English schools do not teach arithmetic in any formal, systematic manner to 5 and 6 year olds, they do achieve a considerable amount of number experience by the end of their second school year. The National Foundation for Educational Research has recently carried out a survey of arithmetic teaching in primary schools in a large and fairly representative county area. The results show that by the time pupils enter their third year of school (age 7) most schools have taught multiplication by 2, 3, 5, and 10 and sometimes by 4. Nearly all schools teach money sums at ages 5 and 6. In both countries children normally have had six years of school by age eleven, but the use of the years for children in the beginning years differs. It may be worth while to re-examine seriously the work of the primary grades in respect to foundational work with numbers and number relations.

4. A fourth factor which might affect the difference in achievement in arithmetic is the preparation of teachers. In terms of academic training the teachers in California have had more years of preparation than have teachers in England. California certifiction requires a four-year college course of

which approximately four-fifths is liberal education and one-fifth professional education. Elementary teachers in England attend a teachers' training college for two years of professional study after leaving their Grammar school (academic secondary school). In addition they have from three to six months of practice teaching and their first year of employment is considered as probationary. The real issue is not so much the number of years of teacher preparation as it is the kind and quality of preparation. Elementary teachers in the United States suffer peculiar limitations in the case of arithmetic. The subject of arithmetic is usually not taught in high school nor is it a part of the program of mathematics in college. Hence, except for short review courses sometimes provided by teachers colleges, elementary teachers have comparatively limited advanced training to teach this subject. They receive instruction in methods of teaching but only incidentally in the content of arithmetic. As a result, the specialized study of arithmetic, for most American teachers, does not go far beyond what they received in the last year of the elementary school. This is in marked contrast to the advanced college preparation of elementary teachers in social studies, science, and English. If therefore, the same degree of difference in achievement in arithmetic as was found in the present study existed at the time the present teachers were in the elementary school, it must be inferred that teachers in England have a better background in arithmetic than do teachers in California. This is not a criticism of elementary teachers nor of elementary schools but rather of college departments of mathematics and of colleges that prepare teachers for failing to provide advanced work in arithmetic to give a higher degree of scholarly competence in this subject. I have in mind the kind of advanced, scholarly treatment such as is provided in a college level book by B. R. Buckingham entitled *Elementary Arithmetic, Its Meaning and Practice*, 1947. There is some evidence that this situation is being improved.

5. As was shown earlier, California children (as well as those in the United States in general) spread the program in arithmetic over a longer period of years than in England. There, the arithmetic program is completed at age eleven and pupils enter secondary school at age twelve. Here, we continue the arithmetic program through grades seven and eight and begin secondary school at age 14. Although the data do not permit comparisons beyond the ages tested, it is conceivable that if the test had been given at age fourteen the differences might not be as great. But while American children are finishing arithmetic in grades seven and eight, English children of corresponding age who enter their grammar schools are covering two years of high school mathematics. The issue is whether England puts too much pressure on pupils or whether California delays its program too much.

6. Finally, the spread of scores on the tests indicates a problem which needs attention. The distribution of scores in Table 3 shows that on the total test 615 English pupils made scores as low as those made by the lowest third of the California group. However, the scores of the top third (1077 cases) of the English group were equalled by only 13 California pupils. It is this failure to produce pupils with high accomplishment in arithmetic that should cause concern, since it is this top third which furnishes the scientists, technicians, teachers, and most of society's leaders. English secondary schools and universities benefit from a superiority of basic arithmetical ability which is not shared by similar institutions here. Two facts may help to account for the wide distribution of English scores, and especially for the predominance of their scores in the upper third of the range. First, most elementary classes in England are "streamed," or sectioned, by ability and in the highest section special effort is made to prepare pupils for the 11 + year examination. Second, it is common practice for the more efficient teachers to be assigned to these "A" sections. In this way the English schools contribute to the special degree of ability

needed by those who will go on to higher education. It is again the old problem of individual differences in which able pupils are not challenged by a school program that matches their ability.

The hard fact that emerges as the result of this study is that in England, following its customary educational procedures, their pupils at age eleven show a two-to-one superiority in arithmetical achievement as compared with pupils in a similarly selected sample in California of the same chronological age and following its customary educational procedures. The question for the public to decide is whether it is satisfied with this outcome and, in case it is not, to decide under what conditions and limitations improvement should be sought. It then becomes the technical problem of the educational profession to discover ways to better the situation and to test as objectively as possible the changes that are proposed. Explosive, off-the-cuff pronouncements by poorly informed critics will help not at all. But if the same degree of serious, competent research were applied to the problem as, for example, is given to research in industry and the armed forces, improvements of major significance should result. It is the duty of research to publish its findings, whether pleasant or unpleasant. Facts in themselves are neutral. Friends of the school cannot afford to ignore them. Although the data for this study were limited to California, there is no reason to assume that the problem is local.

EDITOR'S NOTE. Dr. Buswell's article should be studied carefully and critically. First, consider the results of the two groups tested and then consider the six "Issues to be Studied" which he has so ably presented. A specific age-level pupil was selected so that this becomes the common factor in the comparison. The matching of the two populations is well described and indicates great care as a step in the research. Also, Dr. Buswell has indicated those exercises which are more particularly "British" in context and reference and has excluded these from his important comparisons. He points out that grade-age-placements in the two groups differ and hence are one factor in the final difference. Because of the different programs in the two countries, one would expect the English results to be higher but one is alarmed at the amount of difference between the means of the two groups. The editor is amazed at the differences in standard deviations as shown in Table 2.

Dr. Buswell has pointed to several factors which may have a lowering effect upon the results from the California pupils. The item of mobility is probably greater there than in most sections of this this country and certainly much higher than in England. He also points to the difference in placement of topics but this is fairly standard in the United States. The California results are probably not much different from those which would be obtained from a valid sample representing our whole country.

What shall we do about the situation? Dr. Buswell states, "The issue is whether England puts too much pressure on pupils or whether California delays its program too much." He calls attention to a mode of "ability grouping" in vogue in England and to differences in teacher training as factors. Are we doing as well as we should in these areas? Are our policies of universal promotion and "educating the whole child" factors and if so do we wish to abandom them? Many serious questions arise. Dr. Buswell has given us the results of his research and he goes farther when he asks the questions which our responsible school people should ponder. Do we want better arithmetic in our schools? We can have it if we really want it and are willing to make the effort.

A Penny of 1855

The picture is the actual size of the penny of 1855. Times have changed during the past 100 years. Your pupils might be interested in making some comparisons of events and values covering the 100-year period. Here is an opportunity to combine arithmetic and social studies and language. The editor would like to print a unit that has been developed on this topic. There are many approaches to this. An old account book or day book kept by a family or a merchant might prove interesting. Old school books show prices and types of problems current at the time.

Color as an Aid in Teaching Concepts

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TEACHING ARITHMETICAL CONCEPTS would be a simple task if all children learned alike. It would be a relatively simple teaching situation, for example, if all children responded equally effectively to the same stimulus in learning compound subtraction. One device or one situation would serve the needs of all learners. The fact that all children do not respond in the same manner to all stimuli, and that quantitative concepts are not discovered in the same way by all learners is now accepted by all educators. The thought patterns of children which determine the acquisition of number concepts and arithmetical processes are different in many ways. Teachers are familiar with the reasons underlying these differences in learning, some of which are mental ability, experiential background, previous number concepts, methods used in teaching previous number concepts, insights, ability, interest, aptitude and attitude. For these reasons elementary teachers must be certain that children have all kinds of experiences in dealing with numbers and number processes.

In learning situations, there are some children who must have more concrete experiences than other children. There are children who need several kinds of sensory experiences to discover and to understand certain concepts. Then, there are children, for some reason or another, who grasp new insights and concepts to a better advantage in the mathematical stage. This kind of learning may be true in most situations or on occasions. This paper suggests ways and means of helping children gain concepts and understandings and to make new discoveries in the mathematical phase through the use of color.

In suggesting the use of color in the more abstract stage of arithmetic instruction, the writer feels that certain cautions should be made. It is not to be assumed that in the social phase the concrete or other sensory phases should be neglected. These experiences are essential and necessary. It is the writer's contention, however, that many children can benefit in one way or another from the use of color in the mathematical phase of arithmetic.

The Use of Color in Presenting Compound Subtraction

Let us use compound subtraction (the decomposition method) as one example in the use of color. The decomposition method of compound subtraction is now most commonly used because this method lends itself best to teaching the meaning of subtraction. Although the idea of re-grouping in this algorism is easily objectified, the fact still remains that some children benefit from experiences in addition to the commonly used place value charts, the use of money and with the use of the abacus. In teaching this method of subtraction there are two basic arithmetical concepts which must be stressed if pupils are to understand the process. They must understand place value and they must understand regrouping. Let us use the following illustration.

Green	Red	Blue
3	4	2
−1	2	9
2	1	3

Using color, the ones' column might be blue, the tens' red, and the hundreds'

10

green. Assuming that the pupil has the problem well in mind and focusing attention upon the fact that 9 ones cannot be subtracted from 2 ones, the teacher can help the learners see the process as follows: "Since 9 ones cannot be taken from 2 ones, we can move (not borrow) one ten to the ones' column. The red four is crossed out and replaced by a red three and a red one which has been taken from the red four is placed to the left of the 2, making it 12," as follows:

$$
\begin{array}{c c c c}
\text{Green} & \text{Red} & \text{Red} & \text{Blue} \\
 & 3 & 1 & \\
3 & \not{4} & 2 & \\
-1 & 2 & 9 & \\
\hline
2 & 1 & 3 &
\end{array}
$$

Here the pupil can easily see why the red 1 (ten) and the blue two can be grouped as 12. In other words this method of approach will give correct mathematical meaning to the too frequent practice of simply saying, "nine cannot be taken from 2, so borrow one from the four making it 12, then subtract." By the use of color the ten which is taken from the four is so identified by the color red from the tens' column. Many children can now see from an abstract point of view why it is correct to place the number one to the left of the number 2, making it a 12. It is easy for the teacher and the learners to go back and reconstruct the fact that the red one taken from the tens' column has been added to 2 ones making it 12.

The Use of Color in Teaching "Bridging" in Addition

One of the very first complex processes involving regrouping is "bridging" or "carrying" in addition. This concept can be gained, and new discoveries made by some children through the use of color. For example, before bridging is introduced it can be shown that the sums in the ones' column are placed under the ones' column and the sums of the tens' column under the tens' column. The idea of color shows clearly, blue under blue and red under red. Later when carrying is introduced the excess tens can be

shown in the tens' column, with the appropriate color.

$$
\begin{array}{c c}
\text{Red} & \text{Blue} \\
1 & \\
2 & 7 \\
3 & 8 \\
\hline
6 & 5
\end{array}
$$

The one ten which is "carried" over the two would be the same color as the other numbers in the tens' column.

The Use of Color in the Teaching of Multiplication

If color is used in teaching multiplication, it can clearly be seen what is taking place as each number in the multiplier and the multiplicand is used in the process. It is suggested that only one and two place numbers should first be used in helping children to gain concepts in multiplication. The use of large numbers makes the process too involved and complicated, and basic concepts can be learned equally well through the use of small numbers. Problems such as the following might be used:

$$
\begin{array}{c c}
\text{Red} & \text{Blue} \\
3 & 3 \\
& \times 3 \\
\hline
9 & 9 \\
\text{Red} & \text{Blue}
\end{array}
$$

In this simple problem it can easily be shown that when 3 in the tens' column is multiplied by a 3 that the product will be in the tens' column. In larger products, of course, it may also go into the hundreds column. In this problem, some children will discover the reason that the product, 99, is the same color as 33, the multiplicand.

When simple two-place multipliers are used with two-place multiplicands, an insight will be gained by some children as to why the second partial products are moved one place to the left. In this case the products of 2×2 and 2×3

	Red	Blue
	2	3
×2		2
	4	6
4	6	
Green→5	0	6←Blue

will line up in the red or 10's column because both have been multiplied by a fact in the tens' column. The product of 2×2 (both occurring in the tens' column) will be shown in the hundreds column for reason of the fact that 10 tens are equal to 100. This product will be shown in the appropriate color used for hundreds.

The Use of Color in the Teaching of Division

The most complex of the four basic algorisms is division, this can especially be said of two-place division long division. One of the major difficulties encountered in the long-division type of problems is correct number placement in the quotient. In order for the learner to gain correct understandings of the division process he must gain an understanding of place value in partial quotients and in completed quotients. A correct understanding of place-value is also essential to logical estimation in long division. In helping children to gain understanding in most number processes, it is, of course, better to start with a simple problem.

```
       Green  Red  Blue
         1     1    2
    8 /  8     9    6
         8
       ─────────
         0     9
            ───────
               1    6
```

In this example the learner must see the reason for starting with a partial quotient on the left hand side, or in the hundreds' place. Using color, as in previous examples, the 8 could be green, the nine red and the six blue. The thought patterns might be as follows: "taking 8 into the 8 of 800 (represented by green) the partial quotient of 1 (one-hundred) is shown in green just above

the green eight. We are first starting with 800 on the left hand side, so that we can perform a series of subtractions, from the largest to the smallest number." Children should first, of course, be shown that division is a subtractive process. Many learners will be quick to grasp the concept that if the first partial quotient is green that the completed answer will consist of three places, since green is in the hundreds place. This understanding is, of course, based on the assumption that the learner has gained a concept of place value in previous experiences. The partial quotients which follow the first quotient should be of the color indicated by the place values in the dividend.

Now, there is also the problem of regrouping which must be dealt with in division. The following illustration indicates how color may be used in this process.

```
       Green  Red  Blue
    8 /  7     9    6
```

Again, it is best to use a very simple problem in helping children to gain a rather difficult concept. Children can easily see in this problem that the divisor 8 will not go into seven. It is suggested that this problem be first set up as the previous division problem. That is, in the dividend use a green 7, red 9 and a blue six. Now we come to the problem of regrouping and the problem should be rewritten to indicate 79 tens and 6 ones. Rewriting the problem, we now have in the

```
       Red  Red  Blue
    8 /  7    9    6
```

dividend two numbers indicated as being tens, a red 7 and a red 9, or 79 tens. The 6 in the ones place remains blue as the one has not been changed in the re-grouping process. Now, the learner can see that 8 is divided into 79 tens and the partial quotient of 9 is placed in red above the last digit in 79 tens, or above the nine. Here again, the learner can see that the second and last partial quotient will be blue since there are only two digits in tens place.

The Use of Color in the Teaching of Decimals

In teaching the computing of decimals, the use of color has many possibilities. The whole numbers can be written in black and the decimal fractions in representative colors. That is colors which indicate tenths, hundredths, and thousandths. This should help the learner to acquire some logic in determining the reasonableness of answers. It can readily be seen that in multiplying mixed whole numbers and decimal fractions that the final product will be reasonably near the product of the black numbers (or whole numbers). The value of using color in presenting decimal fractions, may be illustrated by the difficult task of explaining the meaning involved in the "caret" method of dividing a decimal fraction by a decimal fraction.

$$\overset{\text{Black}}{2}.\overset{\text{Red}}{5} / \overset{\text{Black}}{2} \quad \overset{\text{Black}}{9} \quad \overset{\text{Black}}{7} . \overset{\text{Red}}{5}$$

The major problem here, of course, is to explain the moving of the decimal to the right in the divisor, making it a whole number and moving the decimal a like number of places in the dividend. Although this is often explained as multiplying both the divisor and the dividend by the same number, most children do not really understand what is happening in the process which will make the final quotient come out correctly. It can be shown with color that the 2.5

$$\overset{\text{Red}}{2}\overset{\text{Red}}{5} / \overset{\text{Red}}{2} \quad \overset{\text{Red}}{9} \quad \overset{\text{Red}}{7} \quad \overset{\text{Red}}{5}$$

actually retains the same value as 25 (tenths) or 25/10 or 25 (in red). Then it can be shown that 297.5 retains the same value written as 2975 (in red) or 2,975 tenths or 2975/10. This method of retaining the original values of the quantities is advocated by Stokes.[1] The writer believes that this treatment when accompanied by the ap-

[1] C. Newton Stokes. *Teaching the Meanings of Arithmetic.* Appleton-Century Crofts, Inc., 1951. pp. 179–180.

plication of color tends to make this difficult process more understandable for mature pupils. It can be safely said that only the very mature elementary pupils will really understand the process of dividing decimal fractions by decimal fractions. It is a waste of time and effort to try to teach this process to the immature elementary pupil.

Other Uses of Color in the Teaching of Arithmetic

There are many ways of using color in the teaching of arithmetic. It can easily be seen how the application of color could help clarify many processes involving the use of common fractions. The denominator could be of the same color as the total parts of a square or of a circle of whatever might be representing the "whole." The numerator could be of another color, which represents parts of the whole.

Primary teachers might use red for numbers in the tens' column to help supplement other materials used for the teaching of "ten-ness."

In helping children who have difficulty with thought problems teachers could help children with the use of color, to learn to analyze problems. For example, the question or responses called for could be written in one color, and the data that are given to be used in solving the problem written in another color. Through this process children can be helped to see how to read a problem and how to make an analysis. This method could also be helpful in giving children practice in arriving at possible solutions, first with the aid of color and later independently. Since a part of the difficulty in solving problems is because the learner lacks confidence, this type of experience would tend to help some in this respect. The writer is quick to say that this type of crutch should be used only as remediation, and should be discontinued before the learner develops an overdependence upon clues.

Summary

In this article the writer has suggested that color can be used to a good advantage

in helping certain children gain additional insight and meaning in the more abstract or "mathematical" phase of arithmetic instruction. It has been suggested that all children do not respond the same to various methods and devices and that this method will not be equally effective with all children. In some instances it may be helpful and in other teaching situations it may be of much lesser value. Many teachers can undoubtedly use color more advantageously than has been done in the past in the teaching of arithmetic.

A limited number of specific examples illustrating the uses of color in arithmetic instruction have been included in this article. Many more opportunities for using color will be found as arithmetic is taught in classrooms. It is again stressed that the basic aspects of arithmetic instruction should not be neglected. The use of color should only be additional re-enforcement and supplementation. The use of concrete devices, sensory equipment, and experiences in functional problem situations should not be neglected. They should come first. The writer is suggesting that the use of color might open up another avenue of approach in an effort to help some children gain insight and understanding in arithmetic.

EDITOR'S NOTE. Particularly in the early stages of learning a process, the use of colored crayon offers many opportunities to direct the learner's attention to important ideas and principles. Dr. Erickson has shown some of these with the basic computations. In work with fractions, denominate numbers, percentage, and geometry many similar opportunities arise. In fact, colored crayon is probably the teacher's most important aid. But this too can be used to extreme so that its effectiveness is lost. Let us restrict the use of color to those circumstances where it is important to call specific things to the attention of pupils. Dr. Erickson's use of color is very different from the association of color with a specific magnitude as for example red with the number 8 and green with the number 4. If color is restricted to the development stages undesirable associations should not become a mental block to later learning.

BOOKS RECEIVED

Boss, Ernst F. *Aids and Shortcuts in Arithmetic.* Portland, Oregon: Published by author, 1956. Paper 38 p. $1.00.

Offers suggestions to teachers who may wish to help the better students to extend their abilities to different forms of computations, beyond that which is commonly done in the class.

Dunnington, G. Waldo. *Carl Friedrich Gauss: Titan of Science.* New York: Exposition Press, 1955. Cloth xi+479 pp. $6.00.

Scholarly biography of Gauss, one of the three greatest mathematical geniuses of all times.

Lasley, Sidney J. and Mead, Myrtle F. *The New Applied Mathematics* (fifth edition). Engelwood Cliffs, N. J., Prentice-Hall, Inc., 1958. Cloth. 457 pp. $3.48.

This book is designed to offer a maximum course for one year's unit in general mathematics. Provisions are made for individual differences by starred questions, optional exercises, and special sections, inventory tests and follow-up practice exercises, as well as topics of value to consumer—budgets, insurance, banks, savings, investments, installment buying, taxes, intuitive geometry and a chapter on signed numbers.

Games

Game-Kalak. Kalak Game Company, P.O. Box #211, Boston 24, Massachusetts 14B. $4.50 (Price list includes playing board, rules and disc counters.)

This game is one of developing strategy and skill. One cannot say that all children would profit from playing it. Yet after the directions are understood the better student might gain much pleasure. We would not classify it as an arithmetic game. Perhaps simpler adaptations of the rules might be made.

Helping the Non-Learner in Grade One

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I N THE FIRST GRADE at our school we have a child who has no understanding of arithmetic and has not retained anything learned in this subject in the first five months of first grade. The class has covered addition and subtraction up to and including sums of ten.

Her problem seems to be three-fold. First of all the numbers hold no meaning for her. They appear to her *only* as abstract symbols and designate neither quantity nor quality. She has no conception of their serial order and at her best can count in rote order to seven. The number is associated with something you learn in school and is not applied to any other situation. Therefore there is no connection between the symbol 1 and its counterpart one toy, one pencil, etc. Secondly, what little she has comprehended of addition and subtraction have served only to confuse her. The two separate processes are not clear in her mind. Alternately she will add when "take-away" is required and *vice versa*. And finally, she has not developed mature first grade working habits. This is most evident in arithmetic. Her work is careless and done without much thought. She will not take time to think things out, but prefers to work rapidly and erroneously in order not to be behind her classmates. Consequently the arithmetic which she does is incorrect and her writing is barely legible.

I propose to solve this problem by first of all making the numbers meaningful and familiar to her. Then I will undertake to present addition and subtraction concretely, lead into the abstract form and finally integrate the two processes. Throughout all her work I shall emphasize the importance of thinking things out in a careful, accurate manner. As each new process is learned and accomplished I hope to instill in her confidence and pride in these achievements.

Background Investigation

Before beginning any work with the child, I gave her L. J. Brueckner's arithmetic readiness test. She tested two points below normal and the results of this test verified the facts stated in the problem. Although her testing was low, she tested higher in the mathematical phase than the social phase. She had no knowledge of money, calendar and time. Just prior to this, in January, her class was given the Metropolitan Achievement Test. The results of the test gave her a score of 1.1 which was below average for the test and considerably below the average of her class. At this time I also checked her I.Q. and found that she is capable of learning and retaining first grade arithmetic. At the beginning of the school year her eyes were examined and the need found for glasses which she now wears.

It was discovered that her sister, who is in fourth grade, helped her with arithmetic at the beginning of the year. No assistance was given at home before then. Her sister taught her to count and to do simple addition and subtraction. This has been discontinued. I feel that it was a definite liability. In the first place she presented addition and subtraction only in the abstract form, and at the time when the first grade was learning it concretely and was unaware of the abstract. In addition, her sister is not sufficiently skilled in arithmetic to teach it to anyone else. She, too, has difficulty. Therefore much of her help at home was premature and served to confuse her. Perhaps this may have caused part of her problem.

Counting and Numbers

Our first work was done with rational counting (February 14). I wanted the child to think of the numbers concretely and to visualize their quantity. Poker chips were used to represent each number from one to ten. This was done in sequence. As I called out each number the child in turn represented it by that quantity of poker chips. I suggested that we arrange them in the shape of a Christmas tree so we could see the numbers grow. She had no difficulty in counting poker chips above seven although she had missed these numbers previously in rote counting. I then rearranged the poker chips and took them out of sequence. She counted them correctly in this order.

Since she was able to count the numbers from one to ten rationally I decided to find out what concept she had of a number. I selected the number four and asked her what it meant to her. For several minutes she could say only that it was a number. After thinking and talking about the poker chips some more she decided it also could mean four years old, four o'clock and four toys. This is the first that she has said a number could be anything other than a symbol. Next we talked about the terms "more" or "less." We discovered that some of our groups of poker chips represented more than another group and others less. I then questioned her in this manner about our separate groups of poker chips and she replied accurately.

The child became so interested in counting with poker chips that I decided to tell her simply about how people counted in early times. We discussed how counting was done at first on fingers and toes. She said she had ten fingers and we represented them with poker chips. However, she was certain that she had thirteen toes. She took off her shoes, counted five on each foot and said this would make ten, the same number as fingers. She could not add five and five, but got the answer ten by looking at the ten poker chips representing her ten fingers.

Some time elapsed before our next lesson. The child immediately said that she wanted to hear more about old ways of counting. Momentarily we reviewed finger and toe counting. Again she gave an erroneous number of toes—this time it was nine. She counted them and found five on each foot. She added this amount by counting on her fingers and then representing it by ten poker chips. Then I briefly told her about some other ways of counting. She was interested to know that shepherds used to count their flocks with stones. At the end of the discussion she decided that she liked our present day system better because it was easier to count higher.

Now I wanted to see if our experiences with poker chips could be applied to other concrete situations. I had the child take certain numbers of steps forward and backwards. As she walked she counted her steps, no errors were made. The same idea was followed with books and pencils. She brought me different amounts up to and including ten. These materials were put back in smaller groups and both processes were done correctly.

Since rational counting in its simplest form appeared to be understood I thought we could go on with the other stages of counting. We reviewed enumeration. The poker chips were counted in their various groups extending to ten. I observed that the child could estimate groups from one to four now. However, it was necesssry for her to count groups above that. She usually counted by twos to four and then by ones. I noticed that she was often careless in counting chips. She had a tendency to move the chips faster than she counted. It was evident, too, that her attention span was short and her mind often wandered to other things. This fact was stated earlier, and whenever this habit occurred I tried to make her aware of its disadvantages. Next we progressed to identification. The child named the number of blue, red and white poker chips in a group. She also reproduced various groupings for me. Finally she compared the groupings, stating whether they were more or less than others. She found some groups with the same number and I

told her that this meant they were equal. A few minutes remained so we tapered off the lesson by playing "guess what." We each made designs with designated numbers of chips and took turns guessing what it could be. Everytime one of us guessed correctly we received another poker chip.

I have used the above background in counting to make numbers and groups mean something other than rote counting to the child. I believe now that she is beginning to realize that numbers can also represent concrete objects, and that we find them all around us in our everyday life. She seems to enjoy counting things, and is pleased with the idea that she can count them correctly and I can't find a mistake in her work.

We had been quite successful with poker chips in counting. Therefore I thought it was time to use another concrete experience to see if she could apply this knowledge to the unfamiliar. A wire strung with ten beads was selected for our next lesson. The child counted the beads first by ones and then by twos, both correctly. I noticed that, as with poker chips, she was careless in moving the beads although she reached the right total. The importance of counting carefully was stressed. Next she arranged the beads in groups and then told which was more or less, and when it was the same. The carry-over from poker chips was complete and she worked without error or hesitation.

Now we turned our attention to the number chart on the wall. I called out different numbers up to ten out of sequence and she found them. Several times I selected a number between ten and thirty and she found these correctly also. I was interested to see if our number experiences within the past few weeks had helped her as far as rote counting was involved. I was very pleased when she counted up to twenty-two. Before, she had always missed the number eight and had confused the numbers above ten. So far all of our time had been spent on the cardinal meaning of numbers; now I thought it was time to extend to her the ordinal meaning of a number. The child counted the desks in a row and I explained to her that we sometimes say first instead of one, etc. Then as I pointed to each desk she replied first, second, third, fourth or fifth. First this was done in sequence and then I skipped around. As soon as she understood the process her replies were accurate. The lesson was ended by playing the "five game." The child thought of as many things as she could which meant five. There was quite a difference in her scope of ideas as compared with the first time. Following are some of the things she mentioned: five lollipops, five dolls, five feet deep (swimming pool), five o'clock, five pencils, five pieces of paper, five cars, five snowballs.

I had found that the child was able to identify numbers from one to ten correctly on the number chart. Now I wanted her to learn to make the transition from the abstract to the concrete. As she identified the numbers I suggested, I had her make piles of poker chips representing them. Then we worked on reading numbers from ten to fifty. Sometimes I pointed to the number and had her state it, other times I named a number and she pointed to it. She was fairly accurate in this exercise—only missing eleven which she called twelve (she identified twelve correctly) and twenty which she called fifty. We talked about the sequence of numbers and how in each decade another ten is added to the tens column and the ones column goes consequently from zero to nine.

The child told me she had counted ninety stones on a wall driving to school today. In a discussion of this we later decided that since the car had been going fast she hadn't actually counted each one individually, but had estimated them (guessed approximate number by glancing at the wall). This was an important difference that I wanted her to recognize. However, she assured me that she *could* count to 100. Therefore I decided to let her do rote counting again. This time she really surprised me and counted to forty-four, doubling her accomplishment of last time. I thought this was a good achieve-

ment as we had encountered very few experiences with numbers above ten. Since we were counting I asked her to count by twos which she did to ten. She was unable to count by fives or tens.

It seemed to me that her progress in counting was quite satisfactory. The time had come, I felt, to test her ability in an elementary workbook. We used "Ready for Numbers," by Winston. The child did selected pages on numbers from 1–5 which involved counting and comparing groups, recognizing and matching groups with the corresponding figures, recognizing like quantities in pictures, comparison of sizes of things, recognizing groups and writing corresponding figures, constructing semi-concrete pictures of numbers and showing semi-concrete relationships. She worked quickly and with a fair degree of accuracy. However, her carelessness was evident on several occasions. In recognizing groups and matching them with corresponding figures she circled the wrong number; and in recognizing groups and writing corresponding figures she wrote the wrong number. These were both corrected and I feel they were not errors but a case of hasty thinking. I noticed, too, that her numbers and circles were not written neatly. She must still learn the importance of neatness and to take pride in her accomplishments.

At this time the entire First Grade was given a second Metropolitan Achievement Test (March 8). The child's score in arithmetic advanced to 1.7, about average for first grade level in chronological months. However, as was previously true, her score as compared with that of her classmates was much below average, yet more in keeping with her native ability. In the counting part of the test, both cardinal and ordinal, she did well and missed very few. She made a great deal of improvement in this part of the test as compared with last time. In one section of the above she was asked to write down various numbers. The child wrote the numbers correctly, but her writing was hurried and careless. She went completely haywire in the addition and subtraction of

abstract numbers. Very few were correct and her answers were recorded at random.

As a result of this test and my observations of the child's work in the past three weeks it appeared to me that our first problem was almost solved. Counting had become a meaningful process. Before we left this first stage, however, I felt it was important for her to write numbers more accurately and neatly. Without a good background in writing numbers, her problems in the fundamental processes would be doubled. In addition, I wanted to give her some more experiences with a workbook in hopes of improving her work habits before something new was attacked. At this time I had the child practice writing numbers from one to five. Quite often I noticed that these numbers were extended beyond the lines and the numbers were not formed proportionately.

The child was out of school for several days following our number writing. On her first day back she asked me as she came in the door if we were going to play "our game" today. She refers to my helping her with arithmetic at all times as "our game." Later in the day when I told her we would have time she jumped up and down and clapped her hands. We arranged to have it right after the rest period. She was the first one up from rest and very eager to start. One of her classmates came along a few minutes later and asked her if that was what she *had* to do now. The child replied that she didn't *have* to do it, but we did it because it was fun. We started out writing the numbers from six to nine. This time she made a great effort to write them neatly and in exactly the right way. There was such a great improvement in her writing of numbers, as compared with her previous classwork, that it did not look like the work of the same child. It appears definitely to me that she has the ability but must learn to apply herself. She even remembered how to hold the paper with her free hand so her numbers would not "wiggle." She analyzed her own numbers and noticed the slightest mistake when they extended below the line or were

out of proportion. Now she told me, with much enthusiasm, that she had been doing a lot of counting and wanted to tell me some of it. She had counted nine birds in front of her house, nine more days until Spring vacation and eleven lollipops in the store.

We then worked in the Winston workbook with numbers mainly from six to ten. Our work included recognizing and constructing semi-concrete groups, reproducing numbers, what number comes before, and after, and using the numbers from 1–17 in serial order. This was done accurately and the numbers were formed quite well. I noticed that she can identify a group of five at a glance now, and that similar patterns which occur on the same page are only counted once and then compared. The speed of her work has become more rapid, and she is completely engrossed in this work and distractions do not disturb her. She seems to enjoy this workbook and takes pride in it. Her other workbook was full of errors and she realizes that this one has very few.

Addition

Today we approached the next major problem—addition. I felt, at this time, that the child was ready. The numbers are meaningful to her now, she is familiar with groups and she has made improvement in reading and writing numbers. Moreover, she has learned how to think when performing a task and shows interest and determination in her work. Observation of her oral addition with the class and of her class workbook made it clear to me that she had no conception of the idea of addition. Answers to number questions up to and including sums of ten were mostly wrong, and it was apparent that answers had been put down at random. Many of the answers were zero which had not yet been introduced.

First we used such concrete materials as books and pencils. We added one and one, one and two, one and three, and two and two and found their respective answers. Since the child had been so enthusiastic about poker chips, I decided that the best

way to introduce addition would be with poker chips. The child is familiar with them and they are easy to manipulate. We talked about how we had played with poker chips before and decided it would be fun to play a different game. I gave her two poker chips which she put at separate ends of the table. As she moved them together I asked her what happened and she replied that she had two. I told her to do it again and we could watch one and one make two. Then I gave her another chip which she kept apart from the others. As she moved them together she said two and one makes three. We continued with three and one makes four and the child had no trouble. Now I asked her if she could think of another way to make four. She could not think of it so I made two groups of poker chips with two in each group. Then she immediately put them together and said four. Next we did families of five and six and she thought of all the groupings. I explained to her that five and one makes six just like one and five. She found this idea to be true with some of the other families.

Since she was doing so well with concrete material I decided to work with semi-concrete material on the blackboard. I drew one lollipop and one lollipop to make two lollipops (φ and φ makes $\varphi\,\varphi$). Then I represented it with signs since she was familiar with them ($\varphi + \varphi = \varphi\,\varphi$). She did the same thing after I erased it. She was anxious to do it the grown-up way so we represented it abstractly ($1 + 1 = 2$ and $\genfrac{}{}{0pt}{}{\;\;1}{+1}{\rule{0pt}{0pt}}$),
$$\frac{\;+1}{2}$$

counting the lollipops in the semi-concrete drawing each time. We proceeded in this fashion up to and including the four family. Throughout addition both the vertical and horizontal methods were stressed. The child worked independently of my help. Several times she said take-away for + (and) and leaves for = (makes). I could see that there was confusion in her mind between addition and subtraction. Each time this occurred we referred to the drawings or poker chips and found out that take-away wouldn't make sense. From our lesson today it was

evident that much time had to be spent with concrete and semi-concrete material so the child could become familiar with the relationships. My original decision to postpone subtraction until addition was understood was verified.

In our next lesson we learned nothing new in addition, but reviewed what had previously been learned. The child did all this herself and remembered the different ways to make each group. Today the symbol + was not confused with take-away. Since her progress was so satisfactory I decided to test her ability. I mixed up the combinations she had learned and wrote them in abstract form. All of her answers were correct. To further test her comprehension of addition presented this way and as a check on her number ability beyond the four family, I also put 3 + 2 on the board. After a quick glance at the problem, she said she didn't know it. I told her she could use poker chips to find the answer. She arranged a group of two and another of three, put them together and found five. Then she wrote five below her answer on the blackboard. This made it clear to me that she could comprehend addition at a concrete level, even an unfamiliar situation.

The child was able, at our next meeting, to represent poker chips in addition with all combinations up to and including sums of ten. I directed her by asking her to make the different groups for each number (show me different ways to make six, etc.). Occasionally she would arrange the correct grouping but call it a different combination (for 6 and 3, she said 5 and 3 makes 9). After recounting she always corrected herself. These mistakes, I believe, were due to careless counting. She knew the right sum but could not visualize groups above five. She was anxious to progress quickly and did not take time to count her larger groups accurately. Now the child made semi-concrete drawings on the blackboard of combinations through six. Each time a drawing was completed she then put the problem down in abstract form. Some difficulty was encountered with the five and six families in the abstract form. I

encouraged her to use the poker chips and she was then able to transfer this answer to the abstract form correctly. I told her to feel free to use the poker chips as helpers any time she was not sure of an answer.

I now feel that the child has a good background in concrete number combinations through ten. She is gaining confidence in herself and her ability. She seems to be aware of the fact that she is capable of counting accurately. In addition, she is familiar with the poker chips and realizes that she can really do first grade arithmetic when she uses the chips. My objective now is to gradually familiarize her with abstract addition, to encourage her to use poker chips and markers as helpers and to have her slowly drop these crutches when she fully understands the idea of adding things together.

We reviewed concretely all combinations from seven through ten. The child worked with speed and confidence. Then we made semi-concrete drawings of combinations from five through eight. As each drawing was made the child in turn represented this combination in the abstract form. Often she referred to her concrete material as a helper in finding the solutions. Since her progress had been so marked I decided now to review the abstract number combinations through eight. The child got them all correct without the aid of concrete material. She showed a great improvement in the manner in which she worked. She read each problem carefully, thought it out for a few minutes and then wrote the answer in neat, careful writing.

Semi-concrete drawings were made at our next lesson of the numbers six through ten. We progressed with the same system which has been used throughout the learning of addition. This time, instead of using concrete material for reference, the child discovered that she could figure out different family combinations from her original drawing for that family. Recently the child has made only one semi-concrete drawing for a family. Her progress and understanding had developed so well that I thought she was capable of accepting this challenge—

and she did. I feel this is a big step forward, in both initiative and in visualization of groups. A review test, in the abstract form, of all the addition combinations through ten was then given to her. No mistakes were made. An understanding of addition now seems to have been accomplished. The child has learned that adding is putting groups together. She has progressed from adding concretely, through semi-concrete drawings, to the final stage of abstract addition. Not only has knowledge and understanding of addition been achieved, but the child has begun to formulate desirable first grade working habits.

Subtraction and Integration of the Two Fundamental Processes

On to subtraction, our third major problem. The child was anxious to take this next step up the arithmetic ladder. As soon as she arrived at school she asked me if we were going to play the "take-away" game. When I said yes, she was very enthusiastic. We started out by going over the symbols of addition and subtraction. I explained to her that + and add mean putting groups together and usually give us more. Subtract and − mean separating groups and usually give us less. At first we used chairs and pencils for concrete experiences. The child arranged them in groups of twos and threes and then took one or two away. She encountered no difficulty.

Our next step was to present subtraction with smaller and more manipulative concrete material. Again I chose poker chips. Quite a bit of time was spent on this phase. She seemed to understand the idea of take-away and worked extremely well with the concrete material. I think she adjusted better and more quickly to subtraction at the concrete level than she did to addition. Perhaps the background work in counting and addition had made her more familiar with numbers and prepared her to understand new ways of dealing with them. Throughout our next few lessons we worked with subtraction combinations concretely until we reached a remainder of nine. Each time the

child made a semi-concrete drawing for the number family under study and then represented it in the abstract form. The semi-concrete and the abstract were studied with remainders as high as five.

At first semi-concrete drawings were made simply without number symbols and the objects taken away were crossed out ($\varphi \not\!\!\chi$). Then we progressed to the next step of using the symbols in the drawings ($\varphi\,\varphi - \varphi = \varphi$). She seemed to be able to make the transition fairly well from concrete to semi-concrete at first, but had a lot of difficulty progressing from the semi-concrete to the abstract. Frequently she found it necessary to make several semi-concrete drawings or start again and find the answer with poker chips. Finally the task at hand completely overwhelmed her, and I realized that the method I was using was too progressive and was not meaningful enough. The idea of subtraction was much harder for her to understand. I decided that the only thing to do was to start at the beginning again. To progress any farther might destroy her comprehension of addition and lead to complete confusion of numbers.

At this point I would like to mention a few observations and accomplishments during the above period. The child had written $3 - 1 = 2$ in the abstract form. I asked her if she could show me another way to take something away from three, she replied $1 - 3 = 2$. When I told her this was not possible she could not understand the logic. She gave me an addition problem where this is possible ($2 + 1 = 3$, $1 + 2 = 3$). We talked for a long time about how you cannot *take something away* from a number unless that number is larger and she finally grasped the idea. We had not done any actual practice recently with writing numbers. However, most of the child's written work has been at the blackboard and I have stressed the importance of neat numbers. There has been a tremendous over-all improvement in her number neatness and also the size of the numbers has become more mature.

When we started subtraction anew I decided that a different concrete experience

might be helpful. I hoped that something other than poker chips would help to make the relationships more meaningful to her, and would aid her in visualization of the grouping later on as semi-concrete and abstract numbers. Therefore our concrete work was done with horse chestnuts. We concentrated at first on only the two, three and four families. After each number group was separated concretely, the child then represented it in a semiconcrete drawing and finally abstractly. Previously only one drawing had been made for a family. This time a semi-concrete drawing was made for each subtraction combination. This process took longer but I felt that she needed detailed work on each step. Her answers were correct with one exception ($♀♀♀ - ♀♀ = ♀♀$). When the child read her drawing aloud she discovered her mistake and immediately presented it accurately. I don't believe this to be an error, but simply lack of concentration. We spent some time discussing our two words to remember—listen and think! Throughout her work today I noticed that she often referred to her drawings, but did not refer to the horse chestnuts a second time. The semi-concrete seemed to be a sufficient reference for her when making the transition to the abstract.

In our next lesson we reviewed the subtraction combinations through four with no mistakes made by the child. Progress has started. She is beginning to understand the idea of subtraction or take-away. This understanding is apparent in her work and thinking. Observation of her work with the other first graders was noted. There was a tremendous improvement in her workbook. Not only in her computation; but in her neatness, writing and the manner in which she accomplished her work. Of course, she still makes a few errors on each page. The entire first grade was asked to write numbers from 1–100. The child skipped several numbers in the twenties, thirties and fifties. On the whole she performed well, on an equal with some of the class and better than a few.

She is participating more in the oral arithmetic (number games). At first her thinking

procedure was too slow for her to respond in time to the questions. However, now she has speeded up and has been able to give the correct answers several times. I noticed at first that the child was nervous and unsure of herself. When working with me in a quiet room she was able to control her thinking process and do her arithmetic correctly. However, a slight distraction would upset her work. Of course, a normal atmosphere in school is not without distractions. I felt, though, it was best for her to work quietly at first. For this reason it was a long time before the carry-over was noticed with the rest of the class. Gradually our working quarters became more public. Now she can work in a room with distractions and concentrate as well as any average child six years old.

I feel that the child should be commended on her achievements. It should be noted at this point in the paper that the goal for which we are striving in this problem is not perfection, but a genuine understanding of numbers, addition and subtraction. After a complete and lasting understanding, we sincerely hope that perfection will eventually come. At the beginning of this paper the child's poor achievement in school was not only hindering her academically, but socially as well. If I am able to instill in her an understanding of arithmetic, desirable work habits and one outlet for scholastic participation I will feel that my problem has been solved.

Today we worked with subtraction combinations from five through seven. Understanding was evident and progress was made. Each combination of the five family was presented concretely, then semi-concretely and finally abstractly. This system was used for $6 - 1 = 5$ and $6 - 2 = 4$. However, the child said she knew the rest of the family and wanted to just write the numbers on the blackboard. I permitted her to go ahead in this manner. Now that she has grasped the subtraction idea she is anxious to speed up her learning process. The answers were written correctly in the abstract form. This new pattern was followed for the

seven family—all combinations were done concretely, one representative problem was made semiconcretely and all of the combinations were then written abstractly. The child referred quite often to her original drawing, counting and recounting. Her work was correct.

At this time I introduced zero. One of her problems previously had been her association of zero. Zero was given as an incorrect answer for many of her problems. There was great confusion about what it really meant. Therefore I had decided to go ahead with the two fundamental processes and omit zero until the numbers began to hold a real meaning for her. Now I feel that each number means something and that the introduction of zero at this time will not confuse her. Since the idea of zero as a place holder had not yet been discussed in first grade, I introduced zero as meaning not any or none. We experimented with various concrete objects and discovered that "not any" taken away or added to something leaves or makes the same amount. Then we tried it with abstract numbers and still found this to be true. The child seemed to be clear now on this meaning of zero.

Each day at the end of our lesson a little time was spent working with both abstract addition and subtraction problems, including zero combinations. I mixed up about ten problems and the child completed them. This was done to familiarize her with reading signs and using the right process to solve them. A little time spent on this every day should minimize our problem of integration when subtraction is completed.

Some time was now used to review subtraction combinations of six and seven. Then we proceeded to combinations of eight. Horse chestnuts were used for the concrete representation, one semi-concrete drawing was made for the family and then they were shown in the abstract form. The child is beginning to see the relationships. Whenever I present a grouping and ask her to take something away from it, she usually progresses with one and then in sequence goes up to the highest number that can be taken from it.

She selects the numbers to be taken away on her own initiative and receives no help from me. The nine family was treated the same way. However, $9-6=3$, $9-7=2$, $9-8=1$ seemed to be more difficult for her and she made a semi-concrete drawing for each combination. The child figures out each combination as she desires. Concrete materials are there for her to use and she knows that she may use them as freely as she wishes. She seems anxious to apply her knowledge on a more mature level, and often visualizes the problems in her mind and then attacks them abstractly. In observing her working habits I have noticed that nothing is done from memory; she thinks each process out and then records the answer.

At the beginning of our next lesson combinations of eight and nine were reviewed. Special attention was given to the nine family as the child had experienced some difficulty. Next we attacked our last subtraction family, ten. She asked permission to skip the semi-concrete step. She worked with the combinations concretely and then abstractly, including the zero combinations. At the conclusion of this lesson I felt that an understanding of subtraction had been achieved, and that she had attained a fair degree of competency with this process. Only one problem remains now—the integration of addition and subtraction. The child must learn to recognize the sign of the operation and perform it without confusing the processes.

Time was now spent on abstract work with addition and "take-away" occurring one after the other on some occasions, and at various intervals at other times. At first she was careless in noting the signs $(5-5=10)$. Also when the zero combinations were mixed with the others, she became confused and frequently put down zero for an answer $(0+7=0, 5-0=0)$. We went over all of her mistakes concretely and then returned to the abstract and she had no trouble. Another reason for her mistakes, I believe, was that she was accustomed to working with number families. It took her awhile to adjust her

thinking and to attack each new problem independently when they were not arranged in families. After detailed practice she began to have only a few errors, and finally our goal was reached when she was able to do a page of thirty-six problems quickly and accurately.

It now appears that our problem is solved. The child has confidence in her arithmetic and works quickly and with an average degree of accuracy. It cannot be said that she has *mastered* the addition and subtraction facts up to and including combinations of ten. This was not the goal for which we were striving. Our goal was the development and understanding of numbers, and then the application of this to addition and subtraction. This she has acquired. First grade arithmetic is meaningful to her now and she attacks problems with thought, neatness and interest. If she continues along this path then I am sure that one day mastery and recognition will be hers.

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EDITOR'S NOTE. If anyone disapproves of using poker chips as materials for learning arithmetic then such things as checkers or beans or buttons will serve as well. It is assumed that Miss Holinger used chips of the same color. In most classes of thirty pupils one finds one or more pupils who are nearly as slow in grasping the ideas in arithmetic as the pupil described in this paper. Teachers know that progress comes very slowly and that individual help is required. Miss Holinger has given in detail the steps through which her pupil was led. She was quick to sense a lack of understanding and to redevelop concepts. But how much better it is for the pupil to know what she is doing and to see some sense in it than merely to memorize symbols and combinations of them. Some pupils find it difficult to pass from a direct concrete experience to the abstract symbolization of it. Many of them build a mental intermediate step in which they visualize the process but do not need to record semi-concrete symbols. In some first grades one finds "combinations charts" which contain the basic addition and subtraction combinations. These frequently are placed in an obscure part of the room and pupils use them to verify their work.
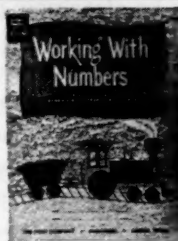
Please mention the ARITHMETIC TEACHER when answering advertisements

Counters? Yes, But . . .

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I T IS GENERALLY ACCEPTED as sound theory that our number system is the result of an evolution of numbers through several progressive stages. It seems probable that primitive man counted his possessions by the aid of his ten fingers; that when he had counted ten and used all his fingers he substituted a stone or other object to which he assigned the value of ten and resumed his counting by his fingers. When he had as many tens-counters (stones, or what had he!) in his tens pile as he had fingers, he again substituted a stone (perhaps a larger one), assigning to it a value of one hundred and resumed his counting by his fingers to more ones, tens or hundreds.

It is regarded as probable also that through stages of refinement there evolved from primitive man's crude beginnings number symbols and the wonderfully orderly number system we know as the decimal system, with its notational system of ten, and place values in the ratio of 10-to-1 (L to R) and 1-to-10 (R to L).

Today's child is far removed from primitive man and his crude efforts at tabulation. In his beginning experiences with numbers he has many opportunities for acquiring the feeling of ten-ness that has been established as the base of our number system. His fingers are handy counters just as were primitive man's. His interest in money and accounting for his money leads him to discover by experience that one penny has a value and that an aggregate of ten pennies is ten, equivalent to a dime, and that he may exchange them for a dime. Later, he discovers the same ten-one relation between dimes and dollars. He is getting the ten-ness idea of numbers that primitive man got with his fingers and ten-stones.

It is at about this point however, that the child's thinking regarding the 10-to-1 ratio of our number system becomes confused. Perhaps primitive man had an advantage over the contemporary pupil in that he had no superiors around to confuse him. No where in accounts of the early beginnings of our number system is it stated that when primitive man had counted ten ones on his ten fingers he bundled his fingers together and called them one ten. Neither is it stated that he piled ten stones together and called them one ten, one hundred, or one thousand. It is, rather, clearly indicated that primitive man thought of a ten as the value assigned to one object which was the equivalent of ten objects, whether ones, tens, hundreds or more where involved in the mathematical operation. He apparently had a clearly fixed idea of the 10-to-1, 1-to-10 ratio "away back when."

In contrast the pupil of today is more often than not confused on the 10-to-1 ratio when dealing with numbers not associated with money. Could the reason lie in the fact that he is confronted with confusing and contradictory concepts? Our money system exemplifies all the basic characteristics of our decimal number system and holds a natural interest and fascination for children. These factors make money an exemplary medium for launching the study of our number system. His early experiences with money can be used as stepping stones to understanding numbers in the same ratio, by the simple method of providing counters with values assigned in the same 10-to-1 ratio he has experienced with money instead of that, however, he is very often confronted with a contradictory concept. He is told that by putting a rubber band around

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ten ones-counters that it has an entirely new value and a new name—one ten. This is entirely at odds with his previous experience of exchanging 10 ones (pennies) for their equivalent value, 1 ten (dime) and so small wonder he is confused. But how easy to avoid confusion by using a counter with an assigned value of 1 ten—a tens-counter. The pupil should be allowed to *exchange* 10 ones-counters for 1 tens-counter. This procedure is not only mathematically correct, but it is also logical and consistent with his previous experiences and concepts. In like man-
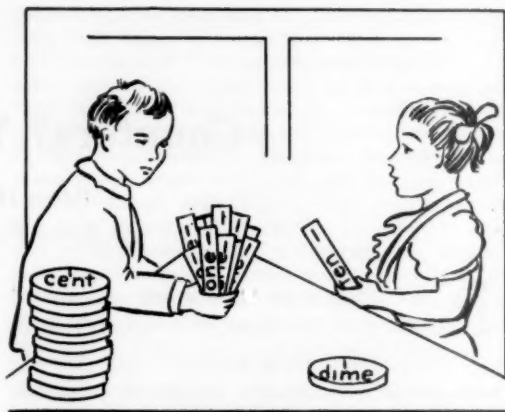
	$2.56			256	
Dollars	Dimes	Pennies(cents)	Hundreds	Tens	Ones
Grouped Values			Grouped Values		
2	5	6	2	5	6
Regrouped Values			Regrouped Values		
	20	56		20	56
	10	156		10	156
1	10	56	1	10	56
2	0	56	2	0	56
		256			256
1	15	6	1	15	6

ner, 10 tens-counters are *exchanged* for 1 hundreds-counter, and so on. Through this manipulation of *exchanging* counters in 10-to-1 (1-to-10) ratio the child is reinforcing and extending his feeling of ten-ness about numbers and at the same time acquiring the concept of grouped and regrouped values of two-place numbers.

The pupil who clearly understands the basic ten-ness of two-place numbers and of the number system and who has had much experience with grouping and regrouping values of numbers eases into the process of regrouping in examples involving the four processes with little or no difficulty. To him it is merely a matter of putting to good use a technique he understands and for which he has developed skill and facility. Take for example a problem involving regrouping in the multiplication of integers, say, 16×3.

He can work it with counters.

Or, on a higher level of operation, writing

tens	ones
16 = {	1 ten	one one one one one one
X 3 = {	1 ten 1 ten 1 ten	one one one one one one one one one one one one one one one one one one
	3 tens	18 ones
Regrouping (10 ones for 1 ten)	1 ten 1 ten 1 ten 1 ten	one one one one one one one one
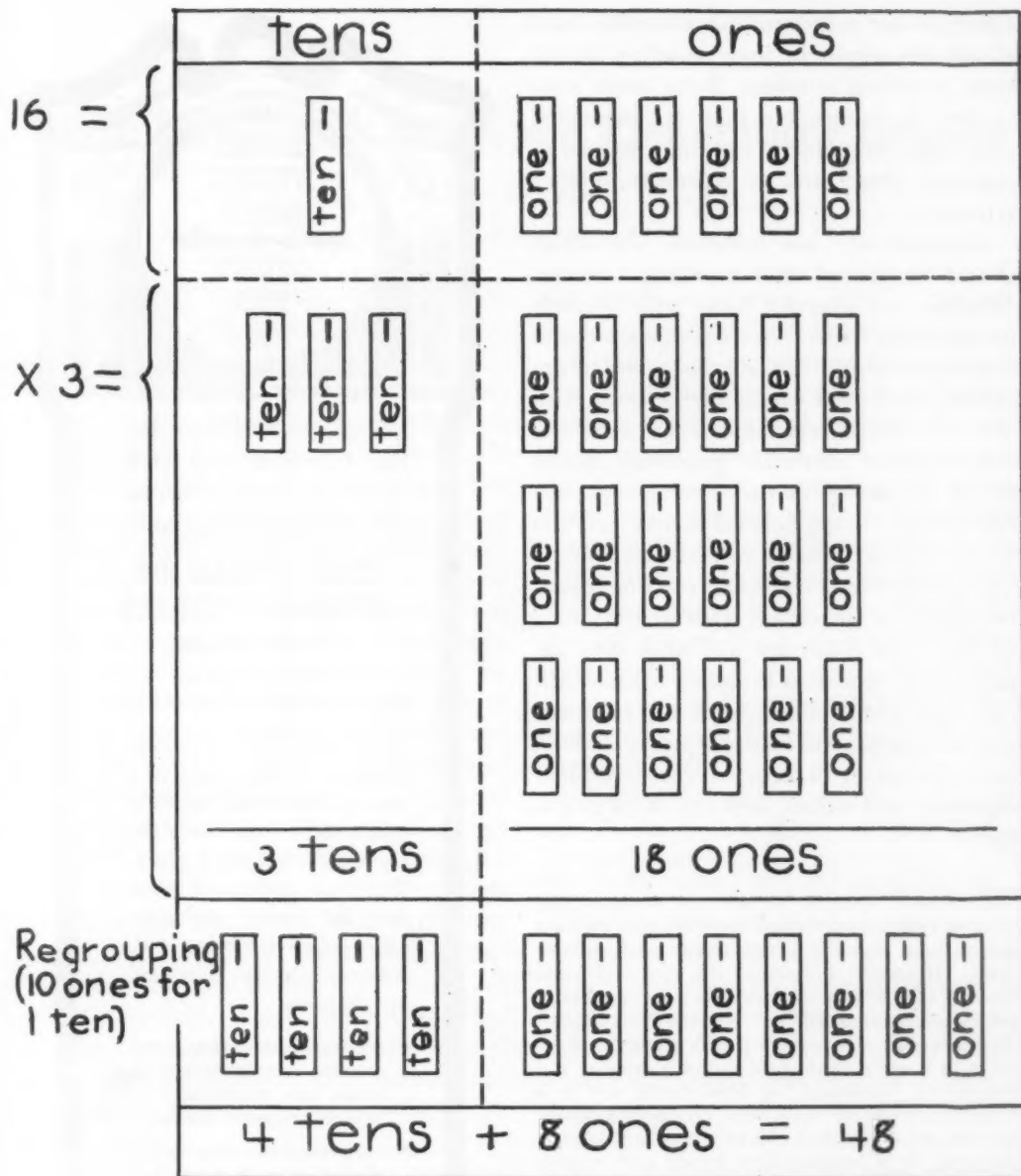
4 tens + 8 ones = 48

out place values and regrouping (or still higher thinking place values and regrouping)

16 = 1 ten and 6 ones

x3   x3

3 tens and 18 ones = 3 tens and 1 ten and 8 ones = 4 tens and 8 ones = 48

Recently in the course of a discussion on counters the objection was voiced that nowhere in life situations is there a parallel to the ones-tens-hundreds-counters suggested here except in money; that in adding books, multiplying pencils, or subtracting cows it isn't possible to show equivalents to groups of ones and so on. The answer to that is the mathematical truth that books aren't added, pencils aren't multiplied and cows aren't subtracted. Numbers symbolic of them are. Number is not a characteristic of things. Numbers are assigned to things in the process of enumeration and computation.

Counters are representative materials used to aid the pupils in understanding operations involving numbers. Being more convenient to manipulate than books, chairs, cows, and other objects they may be used to represent these, and to represent number symbols.

Numbers are not bundled. The child should be spared that bundling concept. Numbers are abstract ideas with symbols for recording them. They have grouped and regrouped values. The fact that numbers are abstract and must eventually be dealt with abstractly is the very important reason why representative materials (counters) should always be shown in their true perspective and should always be used in true relation to our number system: 10-to-1 and 1-to-10. Ten pennies aren't a dime, ten ones-counters aren't a ten either, with or without a rubber band. They are just what they appear to be—ten ones bundled. Mathematical correctness, and consistency in presentation of concepts will help our pupils to avoid confusion about number structure, number meanings and values, and our number system.

EDITOR'S NOTE. Yes, Miss Tuttle, when we work with number symbols we are not working with cows or chairs even though those symbols do have a numerical association temporarily with cows or chairs. If people understand this they will avoid some of the old logical pitfalls such as "the multiplier is always abstract" and "4 ft. $\times$ 3 ft. = 12 sq. ft." And a bundle of ten sticks is still a bundle of ten sticks, it is not a single item except a bundle. The concept of "exchange" as Miss Tuttle uses it is valuable. Let us try to be correct in our description of what we do so that children need not become confused. It is amazing how logical and sensible many very young children can be.

Slide Rules for the Upper Elementary Grades

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THE PUBLIC PRESSURE to produce more scientists will quicken the interest in mathematics. The publicity given to sputnik, the shot truly heard around the world, will resound from the kindergarten through the university. The resulting changes which will undoubtedly occur in the curriculum will have to be evaluated by educators at some future date. But to one who has taught in both public schools and teacher education institutions, it seems apparent that much good will obtain from greater emphasis on mathematics and science. Certainly educators have taken a great deal of criticism (justly given) from lay critics about our mathematically illiterate graduates from both high school and college. In fact, if they are deficient in mathematics on leaving high school, the colleges contribute to this deficiency rather than lessen it. This may happen in one of two ways. First, if they are forced into required college mathematics for which they are unprepared they fail or muddle through thereby increasing their frustrations toward mathematics; or secondly, they avoid mathematics entirely and are four years further removed from it on graduation.

A Tragic Situation

The tragedy of this situation is that many of these people enter teaching and pass their frustrations on to their pupils through their abominable teaching of arithmetic. Many of these fine young people have potential and are capable of doing mathematics. They are good students in other fields. Somewhere along the line they did not receive the inspiration from capable mathematics teachers, or they were guided into competing fields by more enthusiastic teachers or counselors. Therein, perhaps, the teachers of mathematics have failed.

Where does this apparent failure begin? The college teacher bewails at the inadequacy of the high school graduate; the high school teacher bemoans of the poorly prepared junior high graduate; and so on. There is considerable reason to believe that grades five to seven are decisive years in the child's future interest in mathematics. One investigator states it this way, "the pupils' attitude toward the basic science (mathematics) solidified during these years."[1] Looking at the child and the curriculum of these grades it is not difficult to substantiate this contention with logical reasons. First of all the child is changing rapidly himself at this age. He is experiencing such concepts as decimal fractions and percentage, which have been traditionally difficult. Concepts which are often not clearly understood by his parents and older siblings, and which constitute excuses for his own lethargy toward mathematics. Other interests, scouting, athletics, social gangs and the like, begin to take over and occupy his time. If he is to develop a lasting interest in mathematics he needs a great deal of encouragement at this stage of development. This encouragement can come only through a program that can successfully compete with his other interests. One element in such a program which meets the requirements outlined above, i.e., providing the pupil with a sense of growing up while pointing the way to new vistas and a sustained interest in mathematics, is the slide rule.

It seems appropriate, therefore, to bring

[1] *Feel for Science Develops in Youth*, New York Times, February 18, 1957.

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to the attention of the readers of THE ARITHMETIC TEACHER the possibilities of introducing the slide rule to sixth and seventh graders. This article is an account of what might be termed first and second hand experience with a slide rule project inspired by an article which appeared about thirty-five years ago in THE MATHEMATICS TEACHER. First hand as actual experience with sixth and seventh grade youngsters, and second hand, through college arithmetic students who in turn experienced it with their arithmetic pupils.

The detail and construction of the rule used in this experiment are much simpler than the one described in the original article for high school students. There are two considerations which make the simpler rule more desirable. First, it is easier to construct, and second the cost is much less. For about a dollar one can buy enough material to make forty rules. Also a good cheap commercial rule is more available now than thirty-five years ago. Once the child has learned something about the use of the slide rule, the commercial rule will be more desirable from the standpoint of accuracy and greater utility. It should be pointed out, however, that scales other than the one described below are available which will allow considerable latitude for growth for the student who wants to improve the utility of his homemade rule.

Materials for the Homemade Rule

The materials needed for making forty-two slide rules are as follows:

3 pieces of 22″ × 28″ white cardboard @ 20¢ each
4 sheets of single phase semi-logarithmic paper @ 2½¢ each
1 bottle of rubber cement @ 35¢

The three pieces of cardboard are cut in half to make six pieces 11″ × 28″. Three of these pieces are cut into 2″ × 11″ strips and the other three are cut into 1″ × 11″ strips. As shown in figure 1, three strips are required for each rule; one 2″ strip and two 1″ strips. One 1″ strip is attached to the 2″ strip with rubber cement. This adhesive is preferable to others since it prevents warping
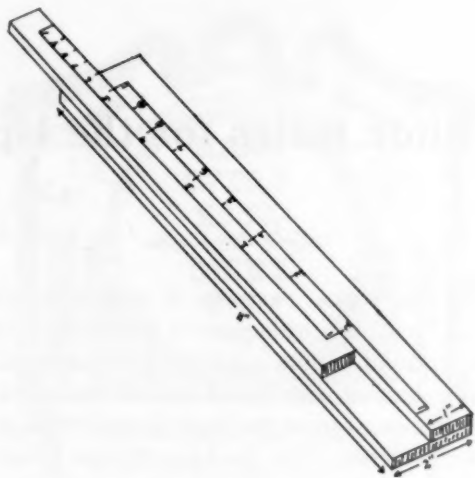

FIG. 1

or buckling, and is much easier and cleaner for the pupils to handle. A paper cutter should be used for cutting the strips so the edges will be straight and will fit and slide smoothly. The logarithm paper should be cut into about three-eighths inch widths (three graph spaces). All preparations can be made prior to class time and the assembling can be done in class in about fifteen minutes. This is particularly satisfying to the child since he can begin learning his slide rule the first day.

The logarithmic strips are pasted to the inner edges of the one inch strips, making certain that the open-spaced end of the graph paper is to the left. Experience has shown thst the rule is a little easier to handle if the movable slide is at the top. A great deal of verbal instructions can be avoided if a rule is made up in advance and placed on a bulletin board nearby so the children can see the finished product.

Learning to Use the Slide Rule Is Easy

It should be emphasized that the teacher should not be afraid to try this project because of a lack of prior knowledge about the slide rule. Actually there is very little to learn about this simple single-scale rule. The greatest difficulty is learning to read the scale. This consists, however, of nothing

more than labeling the main divisions already marked on the graph paper and interpreting the value of the spaces in between. Since the value of the spaces depends on the value given to the large divisions, the rule offers another opportunity to emphasize the concept of place value of numbers. Any division mark, such as, the one labeled 2, can be read as .2, 2, 20, 200 or any multiple of 10. Once the child has learned to interpret the scale the big teaching job has been accomplished. The actual use of the rule to multiply and divide can be fixed in a meaningful way by the use of another aid.

Aids to Learning the Rule

The period of teaching how the rule is read prior to solving real problems can be very exasperating to the teacher. The drill necessary for mastery of the scales can kill the enthusiasm for learning the rule unless the explanations are meaningful and varied. This pitfall can be avoided by the use of teaching aids. Choose a unit that fits your needs (a three-inch unit works well) and make a set of addition sticks three inches wide from heavy cardboard like those used in the primary grades to teach addition and subtraction. Make another set using as your guide the lengths of the major divisions of the logarithm scale (multiplied by 3) for each of the numbers from 2 to 10.

The lesson can then be introduced by demonstrating the use of the unit sticks as they would be used in the primary grades. Lengths 2 and 2 are placed together to make 4; 2 and 3 to make 5; 3 and 3 for 6; etc. A discussion follows about using this principle of adding lengths to multiply. Since the concept of scale development is beyond the pupils' ability to grasp, only the idea is presented, with perhaps a historical note on Napier and his remarkable "bones." The magical set of multiplying sticks is then produced. Lengths 2 and 3 are added to make 6; 2 and 4 produce 8; and 3 and 3 give 9, etc. The question arises about the absence of 1 from the multiplying sticks. The problem is solved when it is recalled that any number times 1 is that number it-

self. Therefore when multiplication is done by adding lengths, no length is necessary for the number 1.

Returning to the addition sticks some subtraction problems are solved with them. The children will reason that subtraction with the multiplying sticks will result in division since addition with them resulted in multiplication. Once this has been accomplished some simple problems should be solved in division with the multiplying sticks. The pupils should then be able to move to the generalization of using two identical sliding sticks graduated in the same ratio as the multiplying sticks instead of the separate sticks. This can be demonstrated by having the reverse sides of the two tens sticks of the demonstration sets marked off as a crude slide rule. Sometimes it is necessary to go over this several times in order that all will see that the slide rule is just a device for eliminating the individual sticks and does quite the same chore of adding or subtracting lengths.

Discoveries from the Demonstration Rule

The class can begin learning the scale from the demonstration rule. Locate the digits on the main divisions between the left index (1) and the right index (10). This will seem to be grasped rather quickly by the pupils; however, the teacher should not be fooled into thinking that they have mastered it. They will need much practice and the resourceful teacher will use as exercises: the child's reproduction of the scale; duplicated scale problems; and pupil checking of individual problems. When they are familiar with the large scale divisions they are ready to solve single-digit multiplication and division problems on their own rules. The range of this type of problem is soon exhausted, so returning to the demonstration rule the mixed numbers $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{4}$, etc., can be located. It is pointed out that the common fractions do not occur on the rule, instead their decimal equivalents can be found. Thus fractional problems like $2\frac{1}{2}$ times $3\frac{1}{2}$ can be given before the two digit

slide-rule-equivalent of 25 times 35. There seems to be at least three advantages to this approach: the child becomes accustomed to working with decimals at once; it is easier initially to think of scale division in fractional terms, i.e., $2\frac{1}{2}$ is easier to locate than 25; and it greatly increases the number of problems that can be solved before the additional difficulty of changing the index is introduced.

To acquaint the child with the principle of considering any reading as a multiple of 10, some problems such as, 2×20, 3×30, 20×20 can be solved. Some exercises in which the left index is considered to be 10 and the numbers 15, 25, 34 and 56 are located will help fix the idea that $1\frac{1}{2}$, 1.5 and 15 are the same point. With this in mind he is ready for the problem 5×3. This of course means changing the index with the slide extending to the left instead of the right as it has up to this time. Since he knows the answer and knows where it is located on the rule he should be able to figure out the index for himself. And he should be encouraged to do so. The thrill of discovery is one of the most satisfying rewards in mathematics. For those who cannot make the discovery by themselves some aids will help. By adding a second scale to the demonstration rule they will be able to see where the answer lies. The extended rule should be marked with indices from 10 to 100.

Some Reasons for Using the Slide Rule

First of all, the slide rule is an interest-getter. It meets the requirement of carry-over to out-of-school activities. To this end it probably has no rival in elementary work. Its mechanical characteristics and the fact that it "short circuits" multiplication and division problems has great appeal to pupils of all ages.

One of the first steps in the solution of a written problem should be the estimation of a reasonable answer. The slide rule is an excellent tool for providing this quick reasonable answer. Thinking is encouraged and real mathematical understanding becomes

an objective in the young problem solver's mind.

In physical science the student is confronted with measurement and the reading of numerous kinds of scales. The experience gained with the slide rule scale will help him in his future work. He becomes familiar with the nature of our number system with the reading and interpreting of the rule.

The teaching of decimal fractions is not an easy assignment. It is not accomplished in a single effort. The slide rule offers another, a fresh approach to decimals. Since operations on the slide rule are independent of decimal consideration, the child is forced to make a reasonable estimate of the answer This is invaluable practice in getting him to think through decimal problems, with the result that absurd answers to such problems become fewer in number. Later the better student will discover some rules for himself about the placement of the decimal point through his experience with the rule. These rules should not be taught, but, he can be encouraged to explore the possibilities. The teacher should accent thinking in all slide rule sessions; this is the instrument of thinking people, scientists, engineers and the like; people who are capable of doing much mental work and who use the slide rule to shorten their computation.

The concept of ratio is difficult for young minds to grasp. On the slide rule it takes on new meaning. A single setting produces a multitude of ratios or equivalent fractions. Conversion from one unit to another, such as, inches to centimeters, is easily done and the student gains new insights and finds new interest in dealing with our complicated system of weights and measures.

The graph paper used in making the rule should provoke new interest in graphs. Students can be told that this is the paper on which a picture of the growth of living things can be plotted, or shown as a straight line.

Any arithmetic teacher should be concerned with the future growth of his students. The basis for the theory of logarithms is a distinct possibility for the accelerated

child. A good approach is to make a table of powers using 2 as a base, thus:

$$2^1 \quad 2^2 \quad 2^3 \quad 2^4 \quad 2^5 \quad 2^6 \quad 2^7 \quad 2^8 \quad 2^9$$

$$2 \quad 4 \quad 8 \quad 16 \quad 32 \quad 64 \quad 128 \quad 256 \quad 512$$

It is explained that the numbers written to the right and above are called exponents or logarithms. By studying the above table two laws of exponents can be discovered, i.e., adding exponents (or logarithms) to multiply and subtracting exponents to divide. Later a table of logarithms can be consulted for checking the above simple multiplication and division problems. The pupils who are given this early insight into logarithms are well on their way to a permanent interest in mathematics.

What About the Rule and Exact Answers?

Teachers in the elementary grades who have been concerned with checking pupils' papers for accuracy are often disturbed on discovering that the slide rule gives only three or four place accuracy. Here again the teacher needs vision of what lies ahead for the student of mathematics. While he has been emphasizing exact answers from absolute numbers he has often overlooked the practical world of measurement. In this realm of arithmetic one deals with numbers with only relative accuracy. Each measurement is a compromise between the instrument used and the accuracy desired. Rounding off measurements becomes a necessity because of the limitations of the measuring stick. What this rounding off means from the standpoint of accuracy can be illustrated to the pupils by an example. For instance, suppose the sides of a rectangle are 3.2″ and 4.4″ to the nearest tenth of an inch. How accurate is the EXACT answer 14.08 sq. in. for the area? Since both measurements have been rounded to the nearest tenth of an inch, it is evident that the limits of accuracy for the first number lies between 3.15 and 3.24, and the limits for the second between 4.35 and 4.44. This places the maximum lower

limit for the area at 13.7 sq. in., and the maximum upper limit at 14.8, each rounded to the nearest tenth of a square inch.

Another approach to the problem of the importance of accuracy in a practical sense beyond the third place can be illustrated by returning to the real meaning of place value. Using a meter stick to represent the thousands' place of the number 1,111, draw a line on the chalkboard 1 meter long. Add to this line one-tenth of a meter for the hundreds' place, one-hundredth of a meter or one centimeter for the tens' place, and finally one-thousandth of a meter or one millimeter for the units' place. The question can then be asked, "How important is the fourth measure when compared to the other three?" The point is made when the chalk has to be sharpened to a chisel point to even represent its length on the board.

In conclusion it seems fair to state that the slide rule offers possibilities for growth in real understanding of mathematics in many areas. At the sixth and seventh grade levels it offers a real challenge to both teacher and pupil alike. For the teacher who tries the experiment there are many pleasant discoveries ahead for him.

Horizontally, Vertically, and Deeper Work for the Fast-Moving Class

GERTRUDE DICK HILLMAN

Fairfield, Conn.

FOR SEVERAL YEARS at the Mill Plain School in Fairfield, Connecticut, we have had an interesting grouping program. One of the big areas of disagreement in the philosophy of this grouping as related to the mathematics classes was whether to enrich at a horizontal level or whether to move vertically toward more advanced work. While this issue was still unsettled, Dr. John Clark at a conference held in November 1955 at the University of Connecticut compounded it further by telling the assembled group to do neither, but to "dig deeper"!

Usually by the end of January of each school year, my "fast" seventh-grade section has completed all the required work in per cent, while the other seventh-grade sections are just starting this topic. The natural question then becomes: where do we go from here? This is where we went in 1956.

We decided to do a school-wide poll. After much discussion of suitable topics, "What Is Your Favorite Dessert?" was chosen. A list of desserts to be included in the survey was selected. Our plans included who would go to each grade or section; how we would conduct the poll in each room; how we would record the information received; and what we would do with the recorded information. On this last item, it was finally decided to represent the results in as many different ways as possible. These included the data recorded in numerical statistical form, in the language of per cent, in graphic form, and in words. When all of this would be accomplished, they then desired to put all the material together in some form so that the entire results could be seen at one time.

A notice was sent well in advance to each teacher that on a particular day at a certain time a member of 7-4 would come to conduct a brief survey. The co-operation of all teachers to permit the time for the poll to be taken was sought and granted. On the appointed morning, and with a feeling of much importance and excitement, the members eagerly went to their assigned rooms. Some of the children had interesting experiences that had little to do with arithmetic per se, especially in the lower grades!

After all the raw material was in, the youngsters were assigned to do the necessary computation for the following groups: Kindergarten through Grade 3; Grades 4 through 6; Grade 7 and Grade 8; faculty; total junior high; and total Mill Plain School. Each group then listed by individual grade the selected desserts with the actual number of votes cast for each dessert and the corresponding per cent of the total votes cast. After this was done, a composite summary was made for each of the above combined grades and then for the entire school. The extreme care in making sure that all per cents of total votes in each grade, each sub-total group, and total for the school came to 100% gave good practice in rounding off. Having to balance the number of votes cast in each grade with the composite number for each sub-total and for the total school encouraged and *required* much checking and re-checking. On the morning when we discovered that the total number of votes for the whole school for each dessert equalled the total number of votes cast by twenty-four sections, a spontaneous round of applause came forth!

The entire data was then printed on a master rex-o-graph sheet and duplicated for each member of the class. When the youngsters saw all the facts in this form, they began

to make some pertinent observations. Thus it was that the idea of an interpretative letter to all the children and faculty members was born.

The actual number of votes cast made ideal material for the teaching of pictographs and bar graphs. The per cents of total votes cast was the perfect opener for the teaching of circle graphs. Of course, this type of graph necessitated a "side trip" into the whole study of angles: their kinds; their drawing and their measuring; and the use of the protractor.

The next-to-the-final step was for the class to interpret the information they had collected. They voted to do this in the form of a letter. Each member studied the data and wrote what meaning he could discover from the statistical information given. After all the letters were read orally, the group worked on a single composite letter using the best observations from all the letters. This was done; the letter was rex-o-graphed; a copy was sent to every faculty member to read to his class.

A final step was to put the best of everything done in connection with the unit on a special bulletin board in the corridor. A member of the audio-visual aids department took a picture of the bulletin board so that the project would be permanently recorded.

Because the author believes that the interpretative letter showed such real depth of perception in getting meaning out of statistical data, she is including it in its entirety.

EDITOR'S NOTE. To dig deeper is a very worthwhile type of education particularly for the more able pupil. He is the one who should develop insight and resourcefulness. Mrs. Hillman has shown how a class, working as a group, developed a unit from which a good deal of arithmetic was derived. The thinking involved in reaching decisions on what to do and how to do it was a type of digging deeper. This type of class work also helps pupils to see arithmetic at work and since the pupils are prime movers in the project they have an added interest which the teacher will capitalize upon for enriching and extending learning. No doubt the whole Mill Plain School became conscious of the work of this particular grade.

March 8, 1958

Dear Mill Plain Faculty and Student Body:

You may recall that several weeks ago, members of the 7-4 mathematics class came to you and asked you about your favorite desserts. As a result of conducting this survey, we discovered many interesting facts. Before we share some of this material with you, however, we should like to thank you for your co-operation in letting us interrupt your classes in order to get our data.

We found that the popularity of many of the desserts depends upon the age-range of the voters. For instance, pudding and Jell-o were quite popular with the younger children, especially kindergarteners through third graders, but in the higher grades hardly anybody voted for these desserts. We decided that this might be due to the facts that younger children eat these foods more often, and that older children have simply grown tired of them.

There is no question as to what is Mill Plain's favorite dessert. Ice-cream, by far, received the most votes, 185 out of 578 votes cast. This represented 32% of all the votes. Another way to put this is to say that about one out of every three persons polled voted for ice-cream. However, for some strange reason, the popularity of ice-cream diminishes in grades 4–6. Is this because children of nine, ten, eleven, and twelve years of age are going out more socially and discovering new desserts at this time? It is also true that ice-cream, coming in so many delicious flavors and capable of being combined in so many interesting ways with other foods, would, appeal to a large number of people.

Second in popularity was pie. However, this dessert does not gain votes in any significant way until the fourth grade. Perhaps mothers would rather give the

younger children ice-cream or Jell-o on a piece of fresh fruit. Pie received 154 votes out of 578 or 27% of all the votes cast.

Although chocolate eclairs ranked third in the voting, we felt that there were so many fewer votes than we had expected. The statistics concerning the eclairs were very interesting. They showed that very few votes were cast for them in the lower grades, but their popularity progressed as we got to grades seven and eight. We felt rather certain that the reason behind this fact was that most of the really young children had never heard of chocolate eclairs, let alone had eaten of these delicious pastries. There were 114 votes cast for eclairs, or 20% of the total votes.

The faculty cast no votes for pudding, Jell-o, or doughnuts. Do our tastes change that much as we grow older, or was the voting the result of the fact that most of the teachers were watching their waistlines? Almost one-half of the teachers voted for ice-cream which contains far fewer calories than most other desserts.

"All others," including sherbet and fresh fruit came in fourth in popularity. Apparently there were still many people who believed that "an apple a day keeps the doctor away." Cake, receiving 6% of all the votes ranked fifth. Too many "cake-mix" cakes? In sixth place were doughnuts, polling 5% of all the votes, and last, but not least, came pudding and Jell-o with 3% of the votes.

It was noted that the per cents of total votes cast for each dessert by the junior high department bore a close resemblance to the per cents of total votes cast by our entire school. We reasoned that this happened, because the junior high enrollment made up about two-thirds of the total school's population, and that such a large number of votes from this group would determine to a large extent the results of the survey.

We have a great deal more information than we have put in this letter. The facts which we have collected have been summarized in many different ways. We put all the material together in a chart called "Here Is Our Original Data," and from this chart we made bar graphs, pictographs, circle graphs, and interpreted these facts into the writing of this letter.

Our tabulation of the original material brought to light some mystifying facts which we have as yet been unable to unravel. For instance, why did nobody in grades 8-1 and 8-5 choose chocolate eclairs as a favorite dessert, and yet in 8-6, 89% of the pupils voted for them? Why does ice-cream show such an amazing increase in popularity between kindergarten and first grade? Perhaps you would like a chance to answer some of these unsolved problems yourselves. If so, just stop by the bulletin board across from Mrs. Hillman's room and study the material that was such fun for us to work on.

Sincerely yours,
GRADE 7-4

Comments on Middle-Grade Arithmetic*

JOHN W. DICKEY

New Jersey State Teachers College, Newark

MIDDLE-GRADE ARITHMETIC has made less change in recent years than has the primary-grade arithmetic; but the gains that have been made are in keeping with the newer philosophy of modern arithmetic. Modern textbooks on the teaching of arithmetic, as well as the materials for the children, have also reflected these major changes in the philosophy, materials and methods. A brief look at the changes in content and methods is worthy of our attention.

An earlier statement of the content of these grades was in terms of the *products* to be learned. The focus of attention was primarily upon the subject matter rather than upon the learner. The fourth grade was concerned with the more difficult aspects of whole numbers; the fifth grade, with the beginnings of the systematic work in common fractions; and the sixth grade, with the more difficult common fractions and the serious study of decimal fractions. The emphasis was upon skills acquired; and much of the

work in problem-solving was disguised drill. The results of standard tests were a valid measure of the learnings. Fortunately, in recent years these requirements have become less difficult and more purposeful for the pupil.

A more recent definition of middle-grade arithmetic is in terms of *concepts* to be learned, *skills* to be mastered, and *problem-solving ability* to be acquired. This definition seems more functional. The focus of attention is primarily upon the learner. This definition places the major emphasis upon the *process*, and a somewhat less emphasis upon the product learned. The teacher must "see both sides of the coin"—the products and the processes. Our culture demands competency as well as interest in the personality of the pupil.

In the area of concepts with whole numbers, some fundamental learnings include the place-value idea as used in addition, in compound subtraction, in the handling of the partial products in multiplication, and in the regrouping with division. The relationships between counting, addition and

subtraction, and multiplication and division are important insights to foster organized learning. The use of the abacus, place-value pockets, and similar manipulative material contribute greatly to the ease of acquiring these insights. The artful use of these materials is a responsibility of the teacher. Unfortunately there is little help on the optimal use of manipulative materials based upon careful research. For the present, the teacher must experiment in her own way.

The concepts of common fractions include the meanings of these numbers as a new set of numbers, and some understanding of the four fundamental operations with them. Of course, the most difficult understanding to acquire occurs in the division process. Many who are preparing to teach have trouble with it. A variety of manipulative materials are now available to promote the growth of these insights. The fraction chart, the flannel board with its cut-outs, numerous drawings in the children's textbooks and the diagrams placed drawn upon the blackboard by the teacher, all should contribute to a fuller understanding of the ideas involved. Again, some thought must be given to the optimum use of these materials, especially for the average child and the slow-learning child. The bright child will need very little work with concrete materials for he readily thinks in terms of abstract symbols.

In the work with decimal fractions, such concepts as the extension of the place-value idea to the right of the units place, the relationships with common fraction and with money, and the comparison with whole numbers in the four fundamental operations, are some of the major ideas to develop. Place-value pockets, fraction charts, and drawings in the children's books and those drawn by the teacher should help in making the work meaningful. It is incorrect to assume that children of this age and maturity profit very little from such learning aids. Teachers in training get helpful ideas from them.

When it comes to building skills nothing seems to take the place of practice. Practice is not drill. Drill reflects an older psychology. The crucial problem seems to be how to practice without merely drilling. No one denies that society requires competency with the skills in arithmetic; and it is generally known that we have not done too good a job in recent years. College freshmen in rather large numbers are no more able than the bottom quarter of eighth-grade pupils in these skills. We have been absorbed with the use of the laboratory approach to build concepts, and the skills have suffered. Quite recently, a renewed emphasis is emerging on the mastery of these skills. Intelligent practice to master these skills requires thinking as a part of the learning.

In problem solving much is known about factors related to success; but very little sound information is available about the most fruitful methods of teaching and learning this ability. The reading ability of the learner, the experiential background, general intelligence, neatness of work, daily practice, habits of visualization and of estimating answers and checking for the reasonableness of the results, the attitudes towards arithmetic, these and many other factors greatly influence the learning of problem-solving. Teaching able children seems to raise no major problems; but the work with the average or the slow-learning pupils challenges the most talented teacher. Perhaps the real art of teaching arithmetic meets its acid test at this point.

In sum, the teacher who understands the many meanings in the arithmetic of the middle-grades, and uses the most modern teaching materials available has gone a long way to doing a good job. If these manipulative materials are wisely used and if practice is sufficient in amount and right in kind, and if the evaluation is broadly conceived in terms of attitudes as well as knowledges and skills, the program of arithmetic will be greatly improved in the middle grades.

Whither Research in Compound Subtraction?

(A Second Communication to the Editor)

J. T. JOHNSON

University of California at Riverside

AFTER READING THE ARTICLE in the October, 1955 issue of THE ARITHMETIC TEACHER entitled, "Comparison of Two Methods of Subtraction," by Gladys and Joel Rheins and the one in the February, 1956 issue captioned, "Whither Research on Compound Subtraction," by J. Fred Weaver, the writer cannot help but feel that there is a misunderstanding some place in regard to the teaching of meanings in connection with the compound subtraction technique.

Reference is made to the Brownell-Moser monograph which was published in 1949. Weaver, in the February article, gives two conclusions from this study as follows:

"(1) For instructional programs in which the teaching-learning situation was mechanical and authoritative, the general tendency was for equal additions to be significantly more efficient than decomposition. This is in agreement with the vast majority of previous research findings which, we have good reasons to infer, were based on data from children and adults who had been taught and had learned compound subtraction mechanically rather than meaningfully.

(2) On the other hand, for instructional programs which emphasized mathematical understandings and related meaningful teaching-learning procedures; for situations in which instruction and learning were evaluated on a broad base which included measures of understanding, of transfer, and of long term retention, as well as conventional measures of speed and accuracy of performance,—in cases such as these, decomposition was found to be preferable and superior to equal additions. This finding gives sound and comprehensive research evidence in the United States to-day to initiate instruction in compound subtraction (in Grade III) on a meaningful basis using the decomposition procedure."

The Brownell-Moser study thus recognizes the fact that the equal additions method is significantly more efficient than the decomposition. That is something to the credit of research. But when they say that they have good reasons to infer that the vast majority of previous research was based on data obtained from mechanically taught compound subtraction rather than meaningfully taught, the writer does not fully see the justification for his inference. This may or may not refer to the writer's dissertation on methods of subtraction. Suffice it to say that this was based on meaningful teaching of the process in both methods as witnessed by the writer who supervised the teaching during two years of the seven years spent in getting data for his dissertation. There seems to be an opinion held by and large by teachers and reflected in the Brownell-Moser study that meanings in compound subtraction can be taught only by the decomposition method. This notion should be corrected. It should be easily understood how this notion has become widespread and this is due to the fact that the majority of teachers use the decomposition method and understand its meaning but they do not understand the meaning of the equal additions method because they have not used it. The decomposition method has no monopoly on meaning. There is a simple meaning that explains this method as will be shown directly.

In the second conclusion when speaking of "instructional programs which emphasized mathematical understandings and related meaningful teaching-learning procedures" the terms are very vague. What is meant by "mathematical understandings" and "meaningful teaching-learning procedures?") Should we expect a third grade child to have mathematical understanding? Let us look into these arithmetic meanings a bit. There are three kinds of meaning in

arithmetic teaching.* The first is structural meaning which has to do with the nature of the number system itself. Place value is one good illustration and the one which relates equally well to both the D† and EA† methods of subtraction so that if meaningful teaching is done here it helps the EA as well as the D method. The second is operational meaning which has to do with the various operational steps in the four fundamental operations with whole numbers, common fractions and decimals and the operations in percentage. This sometimes goes by the name of rationalization. Borrowing or carrying in compound subtraction is a pertinent example of this class of meaning. The steps in this operation differ in the D and EA techniques. The third kind is the functional meaning which has to do with the various applications of all the arithmetic procedures. Here again this meaning applies to both the D and EA methods, so that stressing this meaning which should be done helps one method as well as another.

Now we can pin matters down a bit and put our mental finger upon the meaning which differs in the two methods of subtraction, namely the operational meaning. An interesting feature is presented to us here when we analyze the operation in the two methods. In the example at the left the explanation or rationalization of the D method is generally like this; we cannot take 5 from 3 so we take (borrow) 1 ten from the 6 tens leaving 5 tens and add it to the 3 making 13.

$$\begin{array}{r} 63 \\ 25 \\ \hline 38 \end{array}$$

Then we can say 5 from 13 is 8 and 2 from 5 is 3. Now just what is it that should be understood here by the pupil in order to get the *meaning* out of these explanations to make them seem reasonable to him? It is the understanding of the meaning of place value is it not? And that is a structural meaning which forms the very basis of the understanding of compound subtraction whether it be done by the D technique or the EA

* See writer's more detailed article in *The Math. Teacher*, Vol. XLI, Dec. 1948, pp. 362–67.

† D represents decomposition and EA represents equal addition.

technique. Note in the simple explanation of the EA method which is this, "Add 1 ten to the 3 above so that we can subtract the 5, then to make up for that add 1 ten to the 2 tens below to make 3 tens, then 3 tens from 6 tens leaves 3 tens," that its meaning depends upon the understanding of place value as does the meaning of the D method.

Many operational skills in later arithmetic in multiplication and long division depend upon this basic structural meaning which needs to be stressed more than it is at the present time. We find many 6th grade pupils who cannot write 6-place numbers at the black board from dictation especially if they have some zeros in them. Why is this? Because they do not understand the basic structural meaning of place value.

Incidentally, after the writer had completed his dissertation on the methods of subtraction in 1938 he met teachers on many occasions who argued that the D method was easier to explain than the EA method. This of course would be true with most teachers because most of them were not familiar with the EA method. Some argued that we should use a method that could be used in mixed-number and denominate-number subtraction. It should be known that the EA technique works very efficiently in both of these operations. But the question was intriguing enough to warrant an investigation. A case study of examining several pupils in the 4th and 5th grades was then made by the writer by interviewing pupils individually in several schools in Chicago where the D method had been taught meaningfully and asking the pupil to give a full explanation, out loud, how he subtracted in an example as at left. In nearly every case examined the pupil's response was essentially like this; "I cannot take 5 from 3 so I borrow 1 from the 6 (crossing out the 6 and placing a 5 beside it) and place the 1 by the 3 making 13. Then I can say, 5 from 13 is 8 and 2 from 5 is 3." Not until the writer went into a 6th grade did he find a pupil, a girl, the second or third examined, who gave a rational (meaningful) explana-

tion like this: "As I cannot take 5 from 3 I take one ten from the 6 tens leaving 5 tens and add it to the 3 making 13. Then I can say, 5 from 13 is 8 and 2 from 5 is 3."

The writer carried on the same case study procedure in the 4th, 5th and 6th grades of schools using the EA method and where it had been taught meaningfully but could find none in the 4th or 5th grades who could give him a satisfactory explanation of the meaning of his subtraction, but when he entered the 6th grade, the second pupil examined, a boy, gave this explanation: "Since I cannot take 5 from 3 I add 1 ten to the 3 making 13, and make up for it by adding 1 ten to the 2 tens making 3 tens. Then, 5 from 13 is 8 and 3 tens from 6 tens is 3 tens."

This does not say that no one in the 4th or 5th grades could explain the meaning of compound subtraction by either method as only a small part of each group was examined. We may say, however, that by the current method of teaching meanings in compound subtraction teachers are not getting the meaning fully across to the children until they are older than when first taught and this is very likely to continue with the common run of teachers for a long time. This study was not published as it was not a scientific procedure on the writer's part and proved nothing. He just satisfied his curiosity that the meaningful explanations were for some reasons not well enough understood so that the pupils in the earlier grades could explain them. If an opinion may be ventured from my years of experience with the subtraction question it would be this. The pupils in the 4th and 5th grades do not understand the concept of place value upon which the compound subtraction technique in either method is based and further the teachers are unaware that they do not. If in the near future some teachers will be able to teach the meaning of place value to 4th and 5th grade students we hail the day and both methods of compound subtraction will profit thereby.

The writer agrees with Mr. Weaver that there should be some scientifically constructed and carefully administered and controlled research on the question of compound subtraction and let it be on the question of meaning for we already have abundant research on which is the better method after it is learned. In it the exact words used in the method of explaining the meaning by the teacher should be known and with it a carefully constructed test should be used that would test whether or not the pupil understood the meaning that the teacher explained to him.

Coming now to the last phase of the Weaver article and which is the main reason for writing this article, namely the proposal for teaching the D method first and then changing to the EA method. This, to be frank, was almost a baffling surprise to the writer. It was the spur that caused the writing of this article. Knowing the struggle most youngsters have in learning compound subtraction why should an inferior method be taught and it must be taught to mastery before any meaning can be understood, then followed by unlearning it so as not to produce negative transfer, and then learning another method to mastery to be used for the rest of one's life time? This is equivalent to teaching three methods instead of one. Doesn't one method give enough trouble? A personal experience may be apropos here. The writer in changing from the D method to the EA method to be in harmony with what he advocated took two years to make the change and during this time his check book balance seldom if ever agreed with that of the bank statement.

The writer's advice to teachers in this question is that they should know well the three main methods used by children in this country so that they can help any child that comes to their school by transfer. They should never attempt to change the child's method if he knows his own well. Principals should see that, if the better method is to be taught, pupils start the EA method in the third or fourth grade and let that class use it as they move up through the grades.

Page 20 of the Weaver article should be read carefully by every teacher of arithmetic as it gives some results of research on

three groups who tried the change from D to EA. They were as follows:

The first group who had used the D method and changed to the EA method found that immediately after the change there was a decided loss but after periodic practice after the effect of the negative transfer had been overcome they reached a level significantly above their original ability in the D method. This showed superiority of the EA method.

The second group was one that used the D method but did not change. When their results were compared with those of the group above who had changed (let us call them the D-EA group) they were still significantly above the D-EA group. Does this not show that the change was negative?

The third group used the EA method and did not change. This group to use Mr. Weaaver's own words, "However, Group EA's superiority over Group D-EA's was significant as well as consistent. By the end of the experimental extended period there was no indication that the status of these three groups would change materially with respect to each other, if at all, even with continued practice."

What better research than this can we get? It shows not only that the EA method is superior to the D method but also the negative effect in two cases of the change from D to EA.

The writer agrees with the last three-line paragraph of Mr. Weaver's article if the research be made on, "How early can we teach the meaning of the technique of compound subtraction so that the pupil can understand it?"

EDITOR's NOTE. We are all concerned with final accuracy of results in computations. Subtraction continues to be a part of arithmetic that really bothers many adults. Forgetting of course is human but too many adults, including for a time Dr. Johnson, have too much trouble in balancing accounts and these same adults had high marks in arithmetic in school. What we seem to need is a method coupled with sufficient understanding so that an intelligent adult can think his way through the operation in a reasonable period of time and come through with correct answers. Actually many pupils and adults subtract very well and by different methods and many of us use more than one method

depending upon the figures involved. "Whither Research in Compound Subtraction?" remains a good question. It has many facets. If a rational explanation of the process is to be understood (and remembered) by children certain propaedeutic learnings must be provided. In the method called decomposition it would seem advisable to forget the word "borrow" and use "change" instead. That is implied by decomposition of the number. There is no real borrowing. What shall it be, the more common current method of decomposition or Dr. Johnson's favorite of equal additions? Or, shall we have pupils make their own choice? Or shall we revert to some older method such as "scratch" subtraction? Or, shall we insist that all people carry small computing machines in their purses? "Whither Research in Compound Subtraction?"

Thinking through Problems

Here is a problem with two numbers: How much are buttons a dozen if $1.17 is paid for $3\frac{1}{4}$ dozen?

You have two numbers, $1.17 and $3\frac{1}{4}$. There are only four processes in Arithmetic, $+$, $-$, $\times$, $\div$. Can you add here? No, you can't add buttons and dollars. Can you subtract? No, for the same reason. Very well then, you must either multiply or divide. The answer asked for is money; will the answer be more or less then $1.17? If several dozen cost $1.17, one dozen will cost less than that. So we want a smaller answer. So (since both numbers are greater than one) we must divide.

Another problem: My car goes 15 miles on a gallon of gasoline. How much gas will I need for a trip of 705 miles?

Write the signs of the 4 processes, $+$, $-$, $\times$, $\div$. I can add miles to miles, but I won't get gasoline. Similarly with subtraction. Can we multiply like quantities, dollars times dollars, miles times miles, apples times apples? No. Then we must divide, to see how many sets of 15 miles there are in 705 miles.

Inexperienced teachers may fall into the trap of saying "feet $\times$ feet gives square feet." But logically this is not so. Logically, if a rectangle is 5 feet long, then 5 square feet lie along that side, and if the rectangle is 3 feet wide, there are three columns each of five square feet.

BOOK REVIEWS

Guiding Beginners in Arithmetic, Amy J. DeMay. Evanston, Illinois: Row, Peterson and Company, 1957. Cloth, x plus 178 pp., $2.80.

It is generally agreed that children come to first grade with many number concepts partially developed and with a readiness for systematic instruction leading toward further development. They are curious about numbers and eager to learn more.

Miss DeMay's book provides assistance for the teacher who wishes to capitalize on this zest of learning during the first and second grades in schools. It provides a carefully developed sequence of abilities with definite suggestions for activities which are appropriate to the maturity level of the children. The activities place an emphasis on thinking first with materials comprising the real situation, then with objects representing the situation, next with pictured groups standing for the situation, and finally with abstract symbols is gradual with many intervening activities designed at various levels of maturity.

The teacher who has been out of teaching for some time will find this book a helpful guide in setting forth the newer emphases in teaching arithmetic to young children. These include an emphasis on (1) using groups to help children discover the meaning of numbers below ten; separating, combining and comparing groups to discover facts; using relationships to help children remember and understand; (2) taking maximum advantage of the number opportunities which are a part of the daily life of the child; (3) introducing oral problem solving and the reading of simple problems with carefully controlled vocabulary early in the child's school experience.

The beginning teacher will like the detail, the wealth of specific suggestions and the definiteness and simplicity of the suggestions offered.

In presenting the number system and the facts above 10, Miss DeMay develops in some detail the ideas suggested by Wheat in his "Psychology and Teaching of Arithmetic" and in his later book, "How To Teach Arithmetic." Children are taught a consistent method of using ten to help them think the more difficult addition and subtraction facts with manipulative materials. They are later guided to think the facts abstractly. Miss DeMay is confident that much experience with tens and ones will enable children to learn number meanings and ways of using these meanings to relate their learnings. For example, if 2 and 3 are 5, then 13 and 2 are 15 because 3 and 2 are still 5 (ones) and the "ten" makes the number 10 and 5 or 15.

The experienced teacher will be somewhat disappointed that the book does not give suggestions for working with the more able children and for meeting the needs of slower children. Some teachers will feel that the emphasis on one way of finding answers limits the creative natures of both children and teachers. They will prefer to have children use a wide variety of materials and discover and prove answers in many ways.

EDWINA DEANS

Arithmetic Review, Ray Wall Fisher, Frederick D. Wahl, Blake W. Spencer. New York: Pitman Publishing Corporation, 1956. Paper, iv+76 pp., $1.40.

It was a difficult task for the writer to find something commendable in this work. The outstanding part of the text is the fact that the authors accomplished the objectives outlined in the preface. "Only the most common applications of the mechanics of arithmetic are taken up . . . emphasis has been placed on the development of manipulative skill rather than the reasoning aspect of arithmetic." To this end they are successful.

The organization of the material is logical; it is arranged in a series of lessons, tests, and drills. It would be an easy text for the inexperienced teacher to follow. It contains only 76 pages which are perforated so that they may be extracted easily. The manual is paper bound, of the spiral type, and is inexpensive. It provides an abundance of material ranging from the simplest drills like 6×7 to more difficult ones like adding 14 six-digit numbers and multiplying 4 complex fractions. It must be assumed that the inclusion of so many unreal computations is for the purpose of developing power and facility in performing number gymnastics and contributes nothing to the understanding of the analyzing and synthesizing processes.

The writer believes that the *Arithmetic Review* has several major faults: First, many of its mathematical concepts are incorrect. Witness the following: A decimal is a fraction or part of a whole number consisting of a decimal (.) followed by figures. As a general rule put the larger number on top when multiplying. A fraction can be expressed as a decimal like (.25) or as a decimal fraction like (25/100). In division, express your remainders as fractions. A complex decimal consists of a decimal and a proper fraction.

Secondly, it contains many definitions and statements of principles which are unsound. Witness the following: Zero is not a digit. Numbers are read by their comma names. Annexed zeros change the whole number in multiples of 10. Annexed zeros do not change the value of the decimal. Regarding Equal Addition in subtraction the authors say that the amount borrowed from the minuend is paid back immediately by adding it to the next figure in the subtrahend.

Thirdly, it presents directions for procedures which mathematically are untrue. Witness the following: It is safe practice to extend a decimal with 0's as needed. To measure a number is to find the number of given units it contains. In problems involving division the directions often suggest a solution by cancellation. When a member of an equation is transferred from one side to the other side of the equals, its sign is changed. For example, when B or R is transferred from its basic equation form $(B \times R = P)$ its multiplication sign is changed to division. To divide a whole number ending in one or more 0's, strike out the zero or 0's and move the point as many places to the left in the dividend as there are 0's in the divisor. A mixed number is changed to an improper fraction by multiplying the denominator by the whole number and adding the numerator to the product. When you cancel decimals always place your point before you cancel.

Fourthly, many of its problems are either unworkable or present great difficulties. Witness the following: Find the area of a room 18×20 feet. Express in ratio form $1,2 =$. Divide 12520098 by 52. What number increased by 20% is 660? 14 is 1/8% of what? How many cards each $3'6''$ can be cut from a sheet of cardboard $4'2''$ by $4'6''$?

The reviewer was unhappy to learn that there are still educators who cling to the mechanical development of the subject when so many people like Brownell, Clark, and others have been expounding for years the meaningful approach to arithmetic. When teachers of arithmetic are giving their utmost to the development of the understanding of concepts and operations such a treatment of the subject as presented in this manual is grossly out of step with modern practice.

MARVIN C. VOLPEL

The Direct-O-Percenter, W. H. Glenn Jr. and G. G. Scott. Huntington Park, Calif.: Educational Supply and Specialty Company, 1956, 50 cents.

The "Direct-O-Percenter," is a nomographic device which directly solves the equation $a/b = c/100$ for a, b, or c when the other two are known. In other words it solves percentage problems of Case I, II, and III. The mathematical principle on which the solution depends is the fact of certain proportions that exist when a line from the vertex of a triangle cuts the base and a line parallel to the base.

The "Direct-O-Percenter" consists of a

right triangle *PEG* printed on an 11″×14″ cardboard, and a separate movable scale *AB* which, in the operation of solving a problem, is placed parallel to the base *EG* at the appropriate position above *EG*. There are guide lines parallel to *EG* at half inch intervals (not shown in diagram). A string *PH* is stretched from the vertex *P* cutting *AB* at *C* and *EG* at *F*. Then the *a* of the proportion is *AC*, *b* is *AD*, *c* is *EF*, and 100 is *EG*. (Or *b* and *c* may be reversed.) Base *EG* is divided into 100 equal divisions. Scale *AB* is also divided into 100 equal divisions, though other *AB* scales arbitrarily divided into any number of equal divisions may be used; in fact a foot ruler or a metric scale may be used for *AB*. Supplying other *AB* scales would increase the usability of the instrument.

The usability of the device is improved if it is glued to a piece of masonite, plywood, or stiffer cardboard. For individual pupil use it is of convenient size, though for demonstration purposes a larger model would be more effective. Two significant figures are as much as can be read.

At the junior high level the "Direct-O-Percenter" would be an effective motivation device. It could be used to teach the important fact that physical models can be made to portray the operation of mathematical principles. With properly directed discussion the device could be used to teach a clearer understanding of the meaning of a proportion. In a unit on percentage the best use might be for the checking of the solutions of problems done by other means.

In a plane geometry class the device might well be used to illustrate an application of similar triangles; to review percentage; to emphasize the close relationships of geometry, algebra, and arithmetic; and to encourage students to use their ingenuity to develop other similar devices. Teachers, particularly those who have not formed the habit of using a slide rule, would find the device a real time saver for converting point scores on tests or semester grades to per cent scores.

The "Direct-O-Percenter" sells for 50 cents each, less 10% in lots of ten or more. It may be ordered from Educational Supply and Speciality Company, 2823 Gage Avevue, Huntington Park, California.

EDWIN EAGLE

Arithmetic Games, Enoch Dumas, Fearon Publishers, 2450 Fillmore Street, San Francisco 13, California. Price $1.50.

The booklet presents a variety of games which can be used for *practice* in arithmetic. There are suggestions for both individual and group activities which can be carried on in the classroom; many of the games are adaptations of "old" favorites, such as Bingo and Ring Toss. The material is well organized, presented according to grade levels, and written concisely. The purpose of each game is discussed, necessary materials listed, and directions and descriptions clearly given. In addition, there is a good introduction which discusses the place of games in the arithmetic program and some suggestions on how to use the contents of the booklet.

In using such material, however, it is important for the teacher to recognize the limitations of games in arithmetic instruction. Games provide *one* type of practice in arithmetic; the ones in this booklet primarily stress computational facts and skills, and as such are of limited value. It must also be recognized that *any practice* in arithmetic should follow understanding and not precede it. Practice to be most effective must be varied and as functional as possible. Practice, when planned correctly, can reinforce and extend understanding rather than just provide meaningless repetition. Games can be fun, they can stimulate interest and motivate practice, but their overuse can be wasteful and even detrimental. As with any teaching device, the teacher should know *why*, *when*, and *how* to use *games*. And when she does, this booklet can be of use to her.

EDWENA B. MOORE

Report of the Nominating Committee

The Committee on Nominations and Elections presents its slate of nominees for offices to be filled in the 1958 election. The term of office for the president and the two vice-presidents is two years. Three directors are to be elected for terms of three years.

In making nominations for the three director positions, the Committee followed the directive adopted by the Board of Directors in 1955 which states, "Nominations shall be made so that there shall be not more than one director elected from each state, and that there shall be one director, and not more than two, elected from each region." Members may consult THE MATHEMATICS TEACHER for October, 1955, for a map of the regions as they are now defined.

Since the Southwestern Region will have no carry-over director, it was necessary for the Committee to nominate two persons from that region to avoid making nomination equivalent to election.

The following rule, adopted by the Committee, to determine who shall be declared elected as directors, will be followed: The three nominees receiving the largest number of votes will be declared elected, pro-

vided that at least one of them is a nominee from the Southwestern Region; otherwise the two nominees receiving the largest number of votes will be declared elected, and of the two nominees from the Southwestern Region the one who receives the larger number of votes will be declared elected.

Ballots will be mailed on or before February 19, 1958 from the Washington Office to members of record as of that date. Ballots returned and postmarked not later than March 19, 1958 will be counted.

The Committee wishes to thank the many members of NCTM for their help in giving their suggestions for nominees. It is hoped that all members of our organization will exercise their privilege of voting.

CLIFFORD BELL, *Chairman*
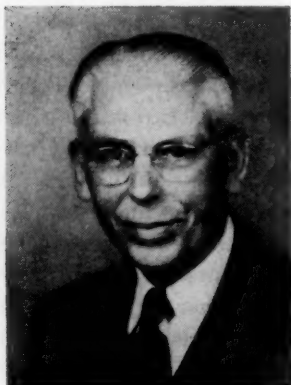JACKSON ADKINS
MARY FOSTER
WILLIAM GAGER
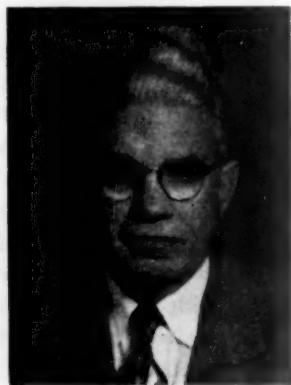ROBERT PINGRY
IRENE SAUBLE
VERYL SCHULT
MARIE WILCOX
LYNWOOD WREN

NOMINEES FOR PRESIDENT

H. GLENN AYRE

HAROLD P. FAWCETT

H. Glenn Ayre

Professor of Mathematics, Head of Department of Mathematics, and Director of General College Division, Western Illinois University, Macomb, Illinois.

Ed.B., Southern Illinois University; S.M., University of Michigan; Ph.D., Peabody College for Teachers.

Instructor in rural schools, Marion County, Illinois; High School, Carterville, Illinois; High School, Benton, Illinois; High School, Waukegan, Illinois; Supervisor of Student Teaching in Mathematics, Professor of Mathematics, and Head of Department, Western Illinois University, Macomb, Illinois.

Member: NCTM and CASMT since 1924; AAAS; NEA; Illinois Education Association; MAA; Phi Delta Kappa; and other educational societies. Listed in *American Men of Science, Who's Who in Education, Who's Who in the Midwest.* Fellow of the American Association for the Advancement of Science.

Activities in NCTM: Vice-President, 1953–55; Chairman, Committee on Affiliated Groups, 1954–56; Program Chairman, Summer Meeting in Seattle, 1954; Editor, Handbook Committee, 1954; Editor, Affiliated Groups Newsletter, 1954; Member, Executive Committee, 1954; Chairman, Committee on Free and Inexpensive Materials, 1953; Member, Algebra Committee for *Seventeenth Yearbook;* Secretary of Symposium on Teacher Education in Mathematics, University of Wisconsin, 1952; Publicity Committee, 1952; speaker and discussion leader for many NCTM programs.

Other Activities: President, Illinois Council of Teachers of Mathematics, 1955–56; Member of Executive Council, 1948–53; Vice-President and Chairman, Illinois Section MAA, 1951–53; Member, Joint Committee of Illinois Section MAA and NCTM to Study Teacher Training in Mathematics; Member, Committee on Tests for ICTM and Illinois Curriculum Project; Chairman, Mathematics Section and Junior College Section CASMT, 1941, 1947; speaker and discussion leader for numerous institutes and workshops in mathematics education.

Publications: "An Analysis of Individual Differences in Plane Geometry," "An Analysis of the Performance of College Freshmen on Arithmetic," *Basic Mathematical Analysis;* numerous articles in professional journals; Coauthor of *A First Course in Coordinate Geometry*, 1956.

Harold P. Fawcett

Professor of Education, Ohio State University, Columbus, Ohio.

A.B. (honors in mathematics), Mt. Allison University, Sackville, New Brunswick, 1914; graduate study, University of California, 1915–17; A.M., Columbia, 1924; Ph.D., Columbia, 1937.

Junior and Senior High School, Fort Fairfield, Me., 1914–15; YMCA Evening School, San Francisco, 1915–17; Field Artillery, France, 1917–19; Home Study Division, YMCA Schools, New York, 1919–24; Instructor, Columbia University, 1924–32; Assistant Professor, Ohio State University, 1932–37; Associate Professor, Ohio State University, 1937–43; Professor, Ohio State University, 1943——: Associate Director, University School, Ohio State University, 1938–41; Chairman, Department of Education, Ohio State University, 1948–56; Visiting Professor, Northwestern University, Columbia University, Universities of Michigan, Virginia, Wisconsin, and Utah; Director of numerous workshops in mathematics education, lecturer, consultant, and educational adviser to Coronet Instructional Films.

Member: NCTM; Ohio Council of Teachers of Mathematics; Ohio Education Association; National Education Association; American Association of University Professors; National Society for the Study of Education; The John Dewey Society; Canadian Universities Associa-

tion; The Torch Club; Phi Delta Kappa; Kappa Phi Kappa.

Activities: Former director of NCTM; Ohio State Mathematics Committee; 1934–35; Committee on Secondary School Curriculum, 1936–38; Committee on Experimental Units, 1942———; Commission on Research and Service, North Central Association, 1943———; Committee on Higher Education, Ohio College Association, 1957———.

Publications: "The Nature of Proof" (*Thirteenth Yearbook* of the NCTM), *Mathematics in General Education* (committee member), and contributions to such journals as THE MATHEMATICS TEACHER, *School Science and Mathematics, Ohio Schools, Educational Research Bulletin, North Central Association Quarterly, The English Journal, California Journal of Secondary Education,* and *The School Executive.*

NOMINEES FOR VICE-PRESIDENT—SENIOR HIGH SCHOOL LEVEL

IDA MAY BERNHARD

RACHEL P. KENISTON

Ida May Bernhard

Consultant in Secondary Education, Texas Education Agency (State Department of Education), Austin, Texas.

A.B., M.A., University of Texas; summer sessions at University of Vermont and Teachers College, Columbia University.

Teacher in Texas Public Schools, 1927–45; Supervisor of Mathematics in Laboratory School, Southwest Texas State Teachers College and San Marcos High School, San Marcos, Texas, 1945–52; attended Duke University Mathematics

Institute, 1946–52; Study Group Leader Duke University Mathematics Institute 1950–52; 1954–55; University of California, Los Angeles, California Conference for Teachers of Mathematics, 1951–57; University of Houston Mathematics Institute, 1952–54; University of Texas Mathematics Conference, 1953; Louisiana State University Mathematics Institute, 1950, 1952–55; Consultant, Texas State College for Women, Mathematics Workshop, 1953–56.

Member: NCTM; NEA; AAAS; NASCD; MAA; Texas State Teachers

Association (Vice-President, Alamo District II, 1951–52; President, Alamo District II, 1952–53); State Honorary Member of Delta Kappa Gamma; Texas State Textbook Committee, 1951; Texas Association of Supervision and Curriculum Development; Texas Association of Elementary Principals; Texas Council of Teachers of Mathematics.

Activities in the NCTM: member of Board of Directors, 1952–55, 1955–58; Southwestern Area Representative for Affiliated Groups Committee, 1952–55; appearance on convention programs; participation on committees.

Publications: "Materials Available for Counseling in Mathematics," THE MATHEMATICS TEACHER, April, 1954. Served as leader of committee which prepared Texas Education Agency Bulletin 548, *Suggestions for Teachers of Mathematics*, 1953.

Rachel P. Keniston

Teacher at Stagg High School, Stockton, California.

A.B., Smith College, Northampton, Mass.; M.A., College of Pacific, Stockton, California.

Member: NCTM; California Mathematics Council; New England Association for Teachers of Mathematics; NEA; California Teachers Association; Delta Kappa Gamma.

Activities: Former vice-president, California Mathematics Council; currently county representative of California Mathematics Council; member NCTM Committee of Supplementary Publications; speaker at numerous mathematics meetings both state and NCTM; group leader (instructor) for a week's course at summer meeting of NEATM in 1955; locally, Chairman of Mathematics Committee for Stockton, California; workshops and summer sessions, College of Pacific, Columbia, Plymouth (New Hampshire) State Teachers College, University of California, Los Angeles.

Publications: Coauthor of textbook: *Plane Geometry;* article in *Eighteenth Yearbook* of the NCTM; articles in *Bulletin, California Mathematics Council.*

NOMINEES FOR VICE-PRESIDENT—ELEMENTARY SCHOOL LEVEL

MARGUERITE BRYDEGAARD

E. GLENADINE GIBB

Marguerite Brydegaard

Associate Professor of Education, San Diego State College.

A.B., San Diego State College; M.A. (and advanced graduate study), The Claremont Graduate School, Claremont, California; travel in Europe to study art and to investigate methods for teaching mathematics in the elementary school.

Associate Editor, *The Arithmetic Teacher*, 1956——; Editorial Board of the Association for Childhood Education International, 1954–56; leader of elementary section, California Conference for Teachers of Mathematics, University of California, Los Angeles; visiting instructor, The Claremont Graduate School; Director of Summer Conferences on Teaching of Mathematics, San Diego State College; led many workshops and presented reports and studies to local, state, and national groups of educators; presented papers at several National Council of Teachers of Mathematics meetings; presented a paper at the University of Wisconsin Mathematics Conference, 1953; worked on elementary-level testing at the Educational Testing Service Workshop, 1955.

Member: National Council of Teachers of Mathematics, Kappa Delta Pi; Pi Lambda Theta; California Mathematics Council; Life Member Association for Childhood Education International.

Activities in NCTM: Member, National Council of Teachers of Mathematics Committee on Supplementary Publications, 1955—, Place of Meeting Committee, 1956——.

Publications: articles include "Creative Teaching Points the Way," first issue, *Arithmetic Teacher;* article in *The Bulletin* (National Association of Secondary School Principals), 1954; coauthor (with Howard Fehr, Leonore John, Ann Peters) *Step Test —Elementary Level;* Coauthor of *Building Mathematical Concepts in the Elementary School,* 1952.

E. Glenadine Gibb

Associate Professor of Mathematics, Iowa State Teachers College, Cedar Falls, Iowa.

B.Ed., Western Illinois University, Macomb, Illinois; M.A., George Peabody College for Teachers, Nashville, Tennessee; Ph.D., University of Wisconsin, Madison, Wisconsin.

High School mathematics teacher, Mendon, Illinois (1941–1945), Geneseo, Illinois (1945–1946); Instructor in mathematics to associate professor of mathematics, Iowa State Teachers College, (1946——); Visiting lecturer, University of Wisconsin (Summer, 1951), University of Vermont (Summer, 1955), Marshall College, Huntington, West Virginia (Summer, 1957); Director of Arithmetic Workshop, Marshall College, 1950, 1951, 1957; Member of institute staffs—Rutgers University (1954), Sedgwick County, Kansas (1954), Louisiana State University (1955).

Member: NCTM; Past member of Board of Directors, Chairman of Policy Committee, Chairman of Elementary Mathematics Section, Central Association of Science and Mathematics Teachers; past vice-president and past president, Iowa Association of Mathematics Teachers; American Educational Research Association; National Education Association; Iowa State Education Association; American Statistical Association; American Association of University Professors; American Association of University Women; Delta Kappa Pi; Pi Lambda Theta; Kappa Delta Pi.

Activities in NCTM: Iowa State Representative; Associate Editor of *The Arithmetic Teacher;* speaker and discussion leader on many convention programs.

Publications: Coauthor of *General Mental Functions Associated with Division and Teaching Children to Divide;* articles in *Review of Educational Research, Journal of Experimental Education,* and *Arithmetic Teacher.*

NOMINEES FOR THE BOARD OF DIRECTORS
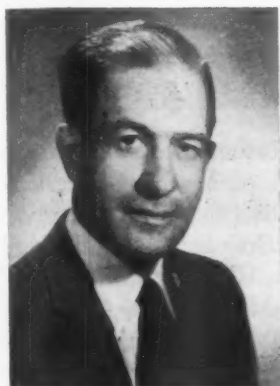
FRANK B. ALLEN   J. HOUSTON BANKS   BURTON W. JONES

EUNICE LEWIS   BRUCE MESERVE   ELIZABETH ROUDEBUSH

Frank B. Allen

Central Region

Chairman of Department of Mathematics, Lyons Township High School, La-Grange, Illinois.

B.Ed., Southern Illinois University; M.S., University of Iowa; additional graduate study at University of Illinois.

Teacher of Mathematics, Sparta Township High School, Sparta, Illinois; Urbana High School, Urbana, Illinois; Thornton Fractional Township High School, Calumet City, Illinois; Trial Judge Advocate and Claims Officer, Army of United States, 1942–46.

Member: NCTM; CASMT; NEA; MAA; Illinois Council of Teachers of Mathematics; American Association for the Advancement of Science; Men's Mathematics Club of Chicago. Listed in *Who's Who in American Education*.

Activities in NCTM: Committee member for mathematics number of *The Bulletin* (National Association of secondary School Principals), May, 1954; Chairman, Secondary School Curriculum Committee of the NCTM, 1955——; Associate Editor of *Mathematics Student Journal*, 1953–54.

Other Activities: President, Men's Mathematics Club of Chicago, 1952–53;

President, Illinois Council of Teachers of Mathematics, 1954–55; numerous speeches and papers at mathematics conferences and teachers' institutes.

Publications: "Mathematics Tomorrow," *NEA Journal*, May, 1957; "Building a Mathematics Program," THE MATHEMATICS TEACHER, April, 1956.

J. Houston Banks
Southeastern Region

Associate Professor of Mathematics, George Peabody College for Teachers.

B.S., Tennessee Polytechnic Institute; M.A. and Ph.D., George Peabody College for Teachers.

Elementary schoolteacher and principal; high school mathematics instructor; high school principal; junior college dean and mathematics instructor; Mathematics Instructor, 133rd Detachment, Army Air Corps; Professor of Mathematics, Florence State Teachers College, Alabama; Visiting Professor of Education, Alabama Polytechnic Institute.

Member: NCTM; MAA; NEA; Tennessee Teachers of Mathematics; Tennessee Academy of Science; TEA; Phi Delta Kappa; Kappa Delta Pi; Pi Mu Epsilon.

Activities: Member, Committee on Supplementary Publications; Southeastern Representative for Affiliated Groups; Speeches and papers at mathematics conferences, National Council, and other professional meetings; Consultant assignments with local school systems; Active in organizing Tennessee Teachers of Mathematics, President, 1954–55; Active part in organizing and conducting state-wide high school mathematics contests; Cochairman, committee on high school contests, Southeastern Section, MAA; Former President, Mathematics Section, TEA; Former State Representative, NCTM; Research in relative effectiveness of various types of mathematics programs.

Publications: Contributor to THE MATHEMATICS TEACHER, *Tennessee Academy of Science Journal*; Author, *Mathematics for General Education* and *Elements of Mathematics*.

Burton W. Jones
Southwestern Region

Professor of Mathematics and Chairman, Department of Mathematics, University of Colorado, Boulder, Colorado.

B.A., Grinnell College; M.A., Harvard University; Ph.D., University of Chicago.

Instructor in Mathematics, Western Reserve University, 1924–26; Instructor in Mathematics, University of Chicago, 1928–29; National Research Fellow, 1929–30; Assistant Professor, Associate Professor, and Professor of Mathematics, Cornell University, 1930–48; Professor of Mathematics, University of Colorado, 1948——; Chairman of the Department, 1949——.

Member: NCTM; MAA; AMS; AAUP; AAAS; Sigma Xi; and Phi Beta Kappa.

Activities in NCTM: Member, Secondary School Curriculum Committee.

Other Activities: Second Vice-President, MAA; Chairman, Committee on Visiting Lectureships, MAA; Regional Consultant in Science and Mathematics, STIP of AAAS.

Publications: *A Table of Einstein-Reduced Positive Ternary Quadratic Forms; Elementary Concepts of Mathematics; Arithmetic Theory of Quadratic Forms; Theory of Numbers;* numerous articles in mathematical journals.

Eunice Lewis
Southwestern Region

Supervisor of Mathematics, University of Oklahoma High School, and Assistant Professor of Education, University of Oklahoma.

A.B., M.A., University of Oklahoma.

Teacher of mathematics at Covington High School, Covington, Oklahoma; Sapulpa High School, Sapulpa, Oklahoma; Cleveland Junior High School and Central High School, Tulsa, Oklahoma.

Member: NCTM; NEA; Oklahoma Education Association; Oklahoma Council of Teachers of Mathematics; Kappa Delta Pi; Delta Kappa Gamma; MAA; Pi Mu Epsilon; Association for Student Teaching.

Activities in NCTM: On program at several meetings of the NCTM; State Representative; Southwestern Regional Representative; Past President of the Oklahoma Council of Teachers of Mathematics; member of committees.

Other Activities: Member of the staff of summer workshops at Louisiana State University, University of Oklahoma, Oklahoma State University, Central State College at Edmond, Oklahoma; Consultant, Oklahoma Study of the Mathematics Curriculum of the State; speaker at local, district, and state meetings of Mathematics Teachers.

Publications: In THE MATHEMATICS TEACHER, "An Experience Program for the Training of Teachers of Mathematics at the University of Oklahoma," "The Pupil Discovers Algebra," "The Role of Sensory Materials in Meaningful Learning." In the 1955 *Association for Student Teaching Yearbook*, "How I Gave My Student Teachers Experience in Establishing and Maintaining Desirable Teacher-Pupil Relations."

Bruce E. Meserve
Northeastern Region

Professor of Mathematics and Chairman, Department of Mathematics, New Jersey State Teachers College at Montclair.

A.B., Bates College; A.M., and Ph.D., Duke University.

Teacher of Mathematics, Moses Brown School, 1938–41; Graduate work at Duke University, 1941–42, 1945–46; Civilian Public Service, 1942–43; U. S. Army, 1943–45; Instructor of Mathematics, University of Illinois, 1946–48, Assistant Professor, 1948–54; Associate Professor of Mathematics, New Jersey State Teachers College at Montclair, 1954–57, Professor of Mathematics and Chairman of Mathematics Department, 1957——; Lecturer at Duke Institute, 1942, New England Institutes, 1952, 1953, 1955, 1956; National Science Foundation Institute at Oklahoma A. and M. College, 1955; New Jersey Institute, 1956, 1957; and at more than twenty other professional groups.

Member: Association of Mathematics Teachers of New Jersey—Executive Board, 1954——, Book Review Editor, 1956——, Committee on Contests; Advisory Committee for the New Jersey Mathematics Institute; Illinois Council of Teachers of Mathematics—Vice-President for Colleges, 1952–54, Chairman of Committee on the Strengthening of the Teaching of Mathematics, 1953–54; Association of Teachers of Mathematics in New England; Executive Committee of New Jersey Section of MAA; American Mathematical Society; Committee on Contests and Awards of Metropolitan New York Section of MAA, 1954–57; Phi Beta Kappa; Sigma Xi; and Kappa Mu Epsilon.

Activities in NCTM: Member of Committee on Research, 1953–54, Chairman, 1954–55; Committee on Institutes and Summer Workshops, 1954–55; Chairman of Committee on the Nomination of an Editor for THE MATHEMATICS TEACHER, 1955; Secretary of Committee on Coordination of Mathematics with Business and Industry, 1954–55, Chairman, 1955–1957; Member of Editorial Committee of *Twenty-Third Yearbook*, 1954–57; Member of Committee on Secondary School Curriculum, 1955——; Chairman, Subcommittee on Secondary School Standards, 1956——.

Publications: Author of *Fundamental Concepts of Algebra* and *Fundamental Concepts of Geometry;* Fourth edition of *College Algebra* (with P. M. Whitman); Co-author of the University of Illinois Bulletin entitled *Mathematical Needs of Prospective Students at the College of Engineering of the University of Illinois;* Author of over twenty articles (five jointly with

other authors) appearing in THE MATHE-MATICS TEACHER and other professional journals.

Elizabeth J. Roudebush
Western Region

Director of Mathematics from Kindergarten through Grade Twelve for Seattle Public Schools, Seattle, Washington.

Graduate of State College of Washington, Pullman, Washington; A.M., Teachers College, Columbia University; summer sessions at University of Washington, University of California at Los Angeles, and University of Southern California.

Teacher, Mathematics, Roosevelt High School, Seattle, Washington; Mathematics Department Head, Edison Technical School, Seattle, Washington; Director of Mathematics, Seattle, 1949——. During World War II served in WAVES as Women's Reserve Representative at U. S. Naval Hospital in Chelsea, Mass.

Member: NCTM; ASCD; NEA; Washington Education Association (Past President of local group affiliated with WEA and NEA); Delta Kappa Gamma (Past Vice-President of local chapter); Pi Lambda Theta (Past President of local alumnae chapter); Soroptimist International (President, Seattle Club).

Activities in NCTM: Member, Board of Directors, 1953–56; Regional Representative of Western Region of Affiliated Groups; Chairman of Place of Meetings Committee; Cochairman in Charge of Local Arrangements for 1954 Summer Meeting; Section Speaker at 1953 Christmas Meeting and 1956 Annual Meeting; Chairman of Affiliated Groups, 1956–59.

Other Activities: helped organize Puget Sound Council of Teachers of Mathematics; helped organize Washington State Mathematics Council.

Publications: *Laboratory Geometry*; "An Arithmetic Bulletin for Parents," THE MATHEMATICS TEACHER, May, 1951; "Professional Classes of the Seattle Public Schools," *Twenty-Second Yearbook* of NCTM.

Minutes

Board Meeting

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
Carleton College, Northfield, Minnesota
August 18–21, 1957

The meeting was called to order at 3:15 P.M., Sunday, August 18, by the president, Dr. Howard Fehr. The following were in attendance.

1. *Philadelphia Meeting.*

MOTION: A motion was made, seconded and passed to approve the minutes of the Philadelphia meeting (March 26–30, 1957) as circulated.

2. *Travel for Board.*

AGREEMENT: It was agreed that the motion pertaining to travel allowance of Board members and the recording secretary under Minute 8 of the Philadelphia meeting be interpreted to mean, in addition to first-class air or train, pullman space and meals en route. If a car is used the basis is to be on first-class air or train fare, whichever is the higher, and room and meals en route. Necessary taxi (or similar service) is also to be included.

3. *Report of Executive Secretary.*

(1) Does not recommend changing the membership files to a punch-card system at the present.

(2) After some delay by the printer the 23rd Yearbook came off the press. Nearly 3000 copies have already been sold.

(3) Conferences have been and are being held with various publishers in order to learn better methods so that errors, expenses and delays may be held to a minimum.

(4) NEA is to provide the Council with more office space.

(5) The filing systems are being revised.

(6) Of the total members: 87 per cent take the *Mathematics Teacher*, 35 per cent the ARITHMETIC TEACHER, and 22 per cent take both journals.

(7) In 1955–56 the Council discarded 25 per cent of its membership plates. Some of this may be due to change of address. NEA loses 17–20 per cent of members each year. Social Studies loses 20–25 per cent. There is a turnover in teachers each year of about 8 per cent.

(8) A financial report was given. This report was circulated.

4. *Dues.*

It was agreed at the Philadelphia meeting that the matter of *dues* would be placed on the agenda of the meeting. After much discussion and thought the following motions were made, seconded and passed:

MOTIONS:

(1) The individual membership dues be raised to $5.00. This includes one journal. Both journals may be had for $8.00.

(2) The institutional subscriptions shall be: (a) The *Mathematics Teacher* $10.00. (b) THE ARITHMETIC TEACHER $7.00.

(3) The student membership dues shall remain the same: $1.50 for one journal and $2.50 for both.

(4) These new rates are to become effective beginning October 1, 1958.

5. *The Mathematics Conference Organization (Formerly Policy Committee).*

Dr. Fehr announced that in addition to himself a second member was being appointed to represent the Council. The new appointee is Dr. John R. Mayor, a past president of the Council.

The original plans have been changed in view of the meeting on April 20, 1957. (See Minute 4 (4) of the March 26–30, 1957 meeting.) It seems now there will be no Institute. The six groups: AMS,

MAA, NCTM, Association of Symbolic Logic, American Statistical Society and SIAM are contemplating forming The Mathematics Conference Organization. Final plans are to be considered at the Pennsylvania State University meeting in late August, 1957. In general the purpose of this organization is to: (1) raise funds for research and projects, (2) promote research and the improvement of teaching, and (3) coordinate work in mathematics. The Board of the organization is to be composed of two members each from the six groups plus six additional members at large. The six members at large are to be selected by the Board itself. Dues to the organization are to be based on two per cent of the membership fees of the individuals to their respective organizations.

Two questions were raised. They are:

(1) The six members at large.

MOTION: A motion was made, seconded and passed that these six should be selected so that there was one from each of the membership groups.

(2) The word *coordinating* as used in the general plan.

MOTION: A motion was made, seconded and passed that the expression "coordinating the mathematical work of the six groups" be changed to read "Promote coordination of mathematical work among the six groups."

MOTION: A motion was made, seconded and passed that the Council would be a part of this new organization for the first year provided the above two changes were adopted by the Steering Committee.

6. Nominating and Election Committee for 1959.

Dr. Fehr asked for recommendations to replace Bell and Wilcox on the committee for 1959. Dr. Fehr will announce the new committee later.

7. Report of the Nominating Committee for 1958.

The report was circulated. The following two motions were made, seconded and passed:

MOTION:

(1) Candidates shall be listed on the ballot in alphabetical order with no regional designation.

(2) In case of a tie vote the Board shall break the tie by means of a secret ballot.

8. Yearbooks.

(1) Jones—24th.
Most of this book has been written. However, there is still too much work to be done in order to release the book by the spring of 1958.

(8) Grossnickle—25th.
Good progress is being made. The book could be released at the Dallas meeting in 1959. However, it would be better not to push but have it ready for Buffalo in 1960.

(3) Johnson—26th.
Progress is being made toward the selection of a committee. Those already selected will have a meeting during this session and plan the approach.

9. Cleveland Meeting—1958.

The meeting will be held April 8–12, 1958. The convention hotel will be Hotel Cleveland. The Board meeting will begin at 1:30 P.M., Tuesday, April 8th. Fehr and Ahrendt will meet with the local committee on November 8–9, 1957. The program, for the most part, has been completed.

10. New York City Christmas Meeting—1958.

The meeting will be held on December 29–30, 1957. The convention hotel will be the Sheraton-McAlpin at 34th and Broadway. The program chairman is Vice-President Robert E. Pingry. Chairman of the local committee is Mr. Kadish, President of the Association of Teachers of Mathematics of New York City.

11. Dallas Meeting—1959.

The program chairman will be the new president of NCTM. The chairman of the local committee will probably be Mr. Harris, who is president of the Greater Dallas Mathematics Association.

12. Greeley Summer Meeting—1958.

Program chairman is Vice-President Alice M. Hach.

13. NEA Cleveland Meeting—1958.

The local organization in Cleveland is being asked to arrange a morning or afternoon session at some central location during the NEA Convention on June 30, 1958.

14. Mu Alpha Theta.

The Council was asked to consider being a sponsor for this group. No action was taken in view of Minute 5 of the Washington Meeting, December 1955.

15. National Institute of Educational Research.

The NEA would like to see such an Institute in the U.S. Office of Education. The Board was asked to consider this project. It was agreed that more information was needed before action could be taken.

16. UNESCO.

The Council was invited to send two delegates to the Sixth National Conference to be held in San Francisco on November 6–9, 1957. Dr. Fehr is to try to locate two members in the San Francisco area to represent the Council.

17. Editor of Mathematics Student Journal.

In view of the resignation of Max Beberman it is necessary to select a new editor. After a study of the many recommendations, Mr. W. W. Sawyer, Department of Mathematics, University of Illinois, was selected.

18. Membership Committee.

The report was received, circulated and discussed.

19. Committee on Relationships with NEA.

The report was received, circulated and discussed.

20. *Elementary School Curriculum Committee.*

The report was received, circulated and discussed. The chairman of this committee wanted to call a meeting of the committee. It was agreed that a complete outline of the project should be prepared and submitted before calling a meeting which would involve expenditures. This outline should be ready for the Cleveland meeting, if not before.

21. *Committee on Place of Meetings.*

Mrs. Marguerite Brydegaard, Chairman, is proceeding with the request to map plans for the next ten-year period.

22. *Committee on Supplementary Publications.*

A report was received, circulated and discussed.

23. *Secondary School Curriculum Committee.*

Dr. Fehr reported that the brochure was being submitted to a selected group of Foundations for consideration.

MOTION: A motion was made, seconded and passed that Dr. Fehr send a letter of complaint to the proper authorities of Teachers College concerning the exorbitant price of their multilith work in connection with the brochure.

24. *Committee on Cooperation with Industry.*

Mrs. Wilcox gave a progress report of this committee.

25. *ICMI.*

See item (12) of the President's report in the minutes of the Jonesboro Meeting, December 1956 and item (6) of the President's Report in the minutes of the Philadelphia Meeting, March 1957. The 1958 meeting will be held in Edinburgh, Scotland. Dr. Fehr will attend and have a prominent part on the program.

MOTION: A motion was made, seconded and passed to allocate $500.00 to be used in sending a delegate to this meeting. The delegate is to be selected at the Cleveland meeting.

26. *Salaries.*

A discussion was held concerning the increased pay scale to be adopted by the NEA in September.

MOTION: A motion was made, seconded and passed to increase the salaries of the clerical staff members in the Washington Office commensurate with the NEA increases.

27. *AAAS Conference.*

Under the sponsorship of AAAS and with Dr. John R. Mayor in charge a recent conference of mathematical and scientific organizations was held. Dr. Fehr gave a report of this Conference. Among the motions passed at this Conference there were two which were directed primarily to the Council. They are as follows:

(1) That this conference refer to NCTM the question of preparing a guide for curriculum studies in mathematics for the use of city and county school systems and state groups. The guide would point out ways in which the curriculums in mathematics can be developed to meet current needs and take advantage of the national interest in the improvement of mathematical institutions at all levels.

(2) That this conference endorse the curriculum study of the NCTM.

28. *Budget Committee—1958.*

Dr. Fehr requested the budget committee to meet and submit a proposed budget to be circulated to the Board at least thirty days before the Cleveland meeting.

29. *Department Officers of NEA.*

Dr. Fehr asked Donovan Johnson to represent him at this meeting. The report of the meeting was received, circulated and discussed.

30. The meeting was duly adjourned at 11:30 on August 18, 1957.

Respectfully submitted,
HOUSTON T. KARNES
Recording Secretary

Thirty-Sixth Annual Convention

National Council of Teachers of Mathematics
April 10–12, 1958
Hotel Cleveland, Cleveland, Ohio

Because of the times in which we are now living, this convention promises to be one of the most significant in our history. Arrangements have been made to provide for a large attendance, and the program provides for every level of instruction from Kindergarten through College each day of the convention. The general nature of the program, described in the following paragraphs, will point to the value of representation at this convention from every school district of the nation.

High lights of the program are the general sessions and banquet. On Thursday evening, at 8:00 P.M., Professor Howard Raiffa of Harvard University will give us a down to earth

introduction to the newest of mathematical developments, the theory of games. At the Friday morning session, Professor John G. Kemeny, of Dartmouth College will speak on "The Discrete in Mathematics," or how simple counting becomes important in todays problems. At the Banquet, Professor S. S. Cairns of the University of Illinois will provide an entertaining and instructive approach to conquering mathematical apathy. The Saturday morning session is honored with the author of "Mathematics in Western Culture," Professor Morris Kline of New York University who will speak on "What Mathematics Shall We Teach." At the Luncheon on Saturday, President Howard F. Fehr will give a brief but snappy talk on "Breakthroughs in Mathematical Thought."

Those interested in the Elementary School will listen to such outstanding speakers as Herbert Spitzer, Glenadine Gibb, Jesse Osborn, Jo McKeely Phillips, Catherine Stern, Nancy Rambusch, John R. Clark, Foster Grossnickle, Marshall Stone, Newton Stokes, Robert Morton, Guy Buswell, and Allan Hickerson. Among the topics these people will treat are the meaning of Symbols, the content and organization of Arithmetic, Meaning in Arithmetic, Ideas in a Fog, the use of materials such as the Stern, Montessori, and Cuisennaire aids to learning, the mathematical structure of Arithmetic, demonstration teaching by radio, and provisions for individual differences. This rich fare can not be missed by any teacher or supervisor or director of elementary arithmetic.

The Junior High School sections provide topics on Curriculum Construction, the Theory of Arithmetic, Building Concepts, teaching problems, the use of models, Modern Mathematics in the Junior High School, challenging content in the Junior High School and provisions for exceptional children. The speakers include Max Beberman, Max Sobel, Robert Fouch, Florence Elder, Henry Syer, Dan Dawson, John Reckzeh, Robert Swain, John Mayor, and Maurice Hartung. All these names have occurred in the Mathematics Teacher from time to time and they guarantee a high type of challenging intellectual stimulation.

For the Senior High School there will be talks and discussions on the Nature and Content of Geometry and of Algebra instruction, Continuity of Concept Learning, Strategy in Concept Formation, the unfolding curriculum in the high school, the use of models, the status and possibility of TV in teaching mathematics, illustration of Modern Mathematics and Statistics for the Secondary School, Programs for the Gifted, and provisions for the less able. The speakers include Frank Hawthorne, Lee E. Boyer, Professor Albert Tucker, Richard S. Pieters, Joseph Hooten, Henry Syer, Phillip Jones, Myron Rosskopf, E. H. C. Hildebrandt, Henry Van Engen and Julius Hlavaty. Teachers will find in these high school sections much that they have been looking for in Modern Mathematics for the high school classroom.

For those interested in College Mathematics and Teacher Training, there will be addresses by Houston T. Karnes, Jack D. Wilson, John E. Freund, Barley Price, Frary Hohn, R. M. Thrall, Bruce Meserve, Karl H. Denbrow, Howard Levi, and Robert C. Yates. This group of outstanding college teachers will discuss the nature of geometry at the college level, Modern Geometry and Modern Algebra for Teachers, Application of Mathematics in the Social Sciences, as well as application of Relational Algebra, College Programs for Freshman Years and for Non-Specialists, and the overall training of Junior and Senior High School teachers for teaching a Modern High School Program.

Among the special features there will be the showing of latest films in mathematics, the meeting of delegate assembly on Thursday from 9:00 A.M. to 12:00 noon, laboratory for Elementary, Junior High School, and Senior High School teachers (with limited registration), a section on Cooperation with Industry, International programs in mathematics and a Youth Forum with Prof. H. J. Ettlinger as speaker.

For guests who come with delegates, as well as the delegates themselves, a number of trips and recreational activities have been planned that will add a gayer moment to the convention. The complete program will be mailed to all members in February. Keep the dates—April 10–12, 1958, and reserve your room—Hotel Cleveland, Cleveland, Ohio.

New Membership and Subscription Dues

M. H. AHRENDT

Executive Secretary

"Because of the increased cost of printing . . . "—these were the words with which an announcement began in the April 1947 issue of the *Mathematics Teacher* giving notice of an increase in dues from $2.00 to $3.00 per year.

During the eleven years that have followed this announcement notable changes have taken place in the Council. The number of individual members has increased about 170 per cent. The number of journals has grown from one to three. An active program of producing supplementary publications has been developed. Distinguished additions have been made to our list of yearbooks. This active program has required more travel by our officers and directors, increased activity by our committees, and an expansion of the clerical staff needed to administer the headquarters office. Meanwhile the cost of printing has continued to increase. Nearly all other costs such as those for clerical help, postage, supplies, and equipment have also increased.

For several years the increasing income from memberships, the active sale of publications, and the attempts of your officers to operate the Council efficiently were able to support the financial needs of the Council in spite of rising prices. We were able to increase our cash reserve each year until 1956. During the 1956–1957 fiscal year, however, our expenses began to exceed our income, with the result that we experienced a deficit of about $8,000.

When the budget for the present fiscal year was adopted, it was apparent that our expenses again would exceed our income, perhaps to the extent of $15,000 or $20,000. Fortunately our cash reserve is still large enough to see us through one more year of deficit operation.

With some reluctance, but with strong faith in the future of the Council and in the loyalty of our members, the Board of Directors voted on August 18, 1957 to adopt the new schedule of membership dues and subscription fees shown below. The present arrangement of making a subscription to either the *Mathematics Teacher* or THE ARITHMETIC TEACHER a part of the membership fee, with both journals available for a slightly higher charge, has been retained.

Individual membership	
Including either journal	$5.00
Including both journals	$8.00
Institutional subscription	
Mathematics Teacher	$7.00
ARITHMETIC TEACHER	$7.00

The student membership fee remains the same as now, with one journal available for $1.50 and both for $2.50. The subscription fees for the *Mathematics Student Journal* also remain unchanged.

These new fees will go into effect on October 1, 1958, and will apply to memberships and subscriptions as follows:

(1) All orders for which the subscription year begins in October 1958 or later.

(2) All new orders received October 1, 1958 or later regardless of the beginning date.

(3) All renewals of memberships and subscriptions that expire with the last issue of the present school year or a later issue.

Recent developments in science and engineering have focused attention perhaps more strongly than ever before in history on the basic importance of mathematics to the modern age. It is urgent in this critical time that the Council continue to receive the loyal support of its members and that its important program be not handicapped by a lack of financial resources.